

A Statistical Definition of Value

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Abstract

Verbal value definitions lead to possible confusion and disagreement in value estimates. A mathematical definition of value is therefore proposed. Value should be defined as “estimates of the parameters of the possible price distribution for the subject property as of a given date.” Identifying the value estimate reported as an explicit central tendency measure such as mean, mode or median adds clarity and interpretability to the valuation. Reporting measures of variation in distributions offers clients valuable information about risk and uncertainty. Reporting on price variance offers valuers a credible defence against professional indemnity claims in cases where sales occur at unusual prices that were low probability outcomes. The definition should also include forecasts of future possible values to help markets incorporate rational expectations and thereby improve efficiency. Difficulties in application come from the fact that the possible price distribution is by definition unobservable so the parameters must be inferred by indirect means. However, focussing on valuation predication errors—which combine price random errors with valuation errors—for particular classes of property will provide clients with the information on risk they need.

Keywords: Appraisal theory, valuation theory, market value, definition of value

Introduction

Value definitions provide the target valuers aim to hit. Different definitions of value can lead to different value estimates. Moreover, value definitions determine appropriate valuation methods so different definitions require different methods.

Courts, academic authorities and practitioners have had differing perspectives on how to define real estate values.

Over the decades courts and professional societies have struggled to clarify what is meant by value by changing the verbiage in value definitions. Despite these efforts, significant failures of valuation practice contributed to large losses by real estate investors in the 1980s and 1990s. These losses have motivated several countries' attempts to tighten standards and clarify methods, including attention to value definition issues. Academic writers have been bothered by lack of clarity and precision in traditional verbal value definitions and by disparities between traditional definitions and basic economic and finance theories of how prices are determined in markets. (Whipple, 1995, Albritton, 1980, 1982) Traditional definitions create confusion about whether valuers' role is to measure and predict market prices or alternatively to define and create price estimates under a set of standardised assumptions.

This paper proposes a statistical value definition that attempts to represent how academic researchers think about value. This statistical definition could make valuations more replicable, provide insights into how to improve valuation methods and add value for clients by improving the valuer's "products."

The plan of the paper is to:

- Review historical origins and current status of verbal value definitions.
- State a proposed statistical value definition.
- Identify implications for valuation methods inherent in a statistical definition of value.
- Review implications for clients and the profession and how the statistical definition can coexist with and supplement traditional definitions.

History and current status of value definitions

A 1999 Appraisal Institute White Paper notes that "breathtaking advances in technology, globalization and securitization" have occurred. "As these forces have taken hold, clients have requested an ever increasing range of services for which real estate appraisers have excellent training knowledge, and skills. Such services may differ greatly from the classic Market Value estimates...which form the foundation of the profession. The increasing demand for varied services ...has led to numerous

problems...” with promulgation of standards and regulation. (Appraisal Institute, 1999:2) The report notes that these difficulties “lead to a proliferation of market value definitions...For example, Market Value under FIRREA is not identical to Market Value in the secondary mortgage market, is not the same as Market Value for the Employee Relocation Council, is not the same as Fair Market Value in various court jurisdiction and is not the same as Fair Market Value as defined in the *Uniform Appraisal Standards for Federal Land Acquisitions*. (Appraisal Institute, 1999:6)

The White Paper notes that “Appraisers themselves learned to diversify their services following a radically reduced need for appraisals in the early 1990s... Services being sought include underwriting, environmental assessment, engineering, site inspections, servicer reviews, tax appeals, due diligence, portfolio analysis, litigation, highest and best use studies, specialty properties and market and feasibility studies.” (Appraisal Institute, 1999:6,7)

The key “market value” definitions used by valuers had their origins in court decisions where there were *involuntary takings* of property. “Market value” was defined in a *non-market* situation, in order to provide the litigants, the courts, and the public with a perception of fairness in the absence of a normal transaction between buyer and seller. The courts sought to describe the circumstances of a *typical* sale, where the hypothetical transaction would be perceived as fair to the landowner and the public agency acquiring the land. These “typical fair market sale” provisions were later adopted as a standard definition suitable for valuations anticipating normal market transactions.

Through the Middle Ages in Europe, a notion of “fair value” existed, enforced by Church authorities. Prices were supposed to reflect a fair value, based on cost of production or some other notion of intrinsic worth. No doubt the importance of the “just price” concept during this period reflected the thin markets and monopoly and monopsony character of small isolated communities dominated by feudal lords. Markets were not well developed, so prices of necessities might have fluctuated too much in the absence of a concept of just price. Remnants of this “fairness” idea came into the market value definitions promulgated by courts, and correctly so in cases where public authorities acquire land through sovereign powers rather than voluntary

bargaining. The courts essentially interpreted fairness to mean a transaction taking place under normal commercial circumstances without price distorting factors. This made takings transactions valuations mimic typical market sales prices.

The 1907 Australian *Spencer v Commonwealth* case ruling on involuntary acquisition of land for military purposes in Fremantle stated:

“To arrive at the value of the land at that date, we have, as I conceive, to suppose it sold then, not by means of a forced sale, but by voluntary bargaining between the plaintiff and a purchaser, willing to trade, but neither of them so anxious to do so that he would overlook any ordinary business consideration. We must further suppose both to be perfectly acquainted with the land, and cognisant of all circumstances which might affect its value, either advantageously or prejudicially, including its situation, character, proximity to conveniences or inconveniences, its surrounding features, the then present demand for land, and the likelihood, as then appearing to persons best capable of forming an opinion, of the rise or fall for whatever reason soever in the amount which one would otherwise be willing to fix as the value of the property.” (Quoted by Whipple, 1995:77)

This definition became the foundation precedent for defining property values in Australia. Note it’s sophistication in pointing out that many issues could influence value including the circumstances of sale as well as characteristics of the property and evaluation of those characteristics by informed buyers and sellers. Note that the judge ignores the actual circumstances—it was a forced sale, after all. Instead the judge says “suppose it sold then, not by means of a forced sale.”

The definition does not explicitly address the purpose of the valuation, although obviously Mr. Justice Isaacs was attempting to define a value useful in arriving at a settlement in a resumption case—he was constructing a hypothetical fair market transaction where no market transaction was in fact present. In doing so, he proposes a kind of “perfect knowledge” standard very much like the information requirements assumed by economists in describing efficient markets.

But what does language like “voluntary bargaining” and “neither of them so anxious” and “cognisant of all circumstances” mean in practice? How motivated is the seller?

How anxious is the buyer? Does “cognisant” mean of the current state of the property or does it mean in addition, awareness of possible development potential and alternative uses? It gets a bit messy in application because real markets often do not conform to these ideals. Different analysts might reach different conclusions because of the lack of clarity in how to operationalise these standards. (See Albritton, 1980)

The 1909 California “Heilbron case” defined value as:

“The highest price estimated in terms of money which the land would bring if exposed for sale in the open market, with reasonable time allowed in which to find a purchaser, buying with knowledge of all the uses and purposes to which it is adapted and for which it was capable of being used.” (Appraisal Institute, 1992:21) The Appraisal Institute’s 1992 10th edition notes that “Market value is inherently a simple concept—it is an objective value created by the collective patterns of the market—but the definition of market value is controversial. Debate on the subject continues and often centers on rather fine distinctions.” (Appraisal Institute, 1992:18)

“Current definitions of market value reflect different schools of thought on five key points:”

- cash sale or financing terms
- specified property rights
- price versus highest price
- most probably price versus highest price
- equilibrium value versus market value

(Appraisal Institute, 1992:18)

The 12 editions of the American Appraisal Institute textbook and various other professional societies, lawmakers and courts have sought to improve valuation standards by debating and modifying the language in value definitions. Clarification and delegation of responsibility to the valuer’s judgement and data gathering occur in updated versions of these various incarnations of verbal definition of value.

A 1975 RICS definition reads:

“The best price at which an interest in property might reasonably be expected to be sold by private treaty at the date of the valuation, assuming a willing seller, a reasonable period within which to negotiate the sale, taking into account the nature of the property and the state of the market, values remaining stable throughout the above period, the property being freely exposed to the market and no account taken of any additional bid by a special bidder.” (Whipple (1995:77))

This definition raises the “best price” “highest price” “most probable price” issue that has been debated in the literature and courts (see Nevada case cited below). Properties generally sell to the highest offer, but Professor Ratcliff, cognisant of the fact that various “highest” bids might be made and accepted preferred “most probable price” to reflect the most likely successful bid rather than a less likely higher possible bid. While it is conceivable that a property might sell for a higher price, it might be unlikely, so “best price” could lead to a biased valuation estimate. (Ratcliff, 1972)

A Nevada appeals court overturned a verdict in a condemnation case based on its conclusion that “most probable price” in jury instructions was not equivalent to “highest price” saying “in the case at bar, the landowners misused and abused the “highest price” instruction in their closing argument to justify the five million-dollar difference in value.” The court noted that “Neither ‘highest’ nor ‘most probable’ is necessary to the definitions: ‘Fair market value is generally defined as the price which a purchaser, willing but not obliged to buy, would pay an owner willing but not obliged to sell, taking into consideration all the uses to which the property is adapted and might in reason be applied.’”

In parentheses the decision goes on to remark caustically that “(As an aside, the Nevada Court once mistakenly substituted the word “adopted” for “adapted” in the definition and repeated the (originally typographical) mistake so often that one started to doubt the Court’s collective literacy.)” (<http://www.nevadeaindex.com/price.htm>)

Valuation is an odd profession in that its key working concepts have been defined not by valuation professionals but rather by courts without special valuation expertise. It is as if physicists were forced to proceed with research on nuclear processes while relying on a lawyer’s definition of the atom.

Seemingly innocuous fine points of language can make a difference. One of the American value definitions requires the valuer to assume “no special buyers.” I sold a property to a special buyer—The Nature Conservancy—an organisation whose valuation method differs from that of other buyers. TNC considers rare plant and bird species rather than timber or housing sites typical of other buyers’ valuations. The transaction never would have taken place, and certainly not in the form it did (involving a conservation easement) without this special buyer’s special pricing considerations. The “no special buyers” valuation produced an irrelevant price estimate.

Each buyer can be expected to have a different hedonic pricing function, that is, different things matter to different buyers. People with children worry about schools more than people who don’t have children, people with boats like to be on the water, rich people can pay more than poor people, some investors are more risk averse than others, and so on and on through hundreds of buyer, property and neighbourhood and circumstances of sale characteristics that may influence prices paid in particular transactions. This diversity and heterogeneity in real estate transactions, along with infrequent transactions, are the reason for the existence of the valuation profession—if property were a fungible commodity like wheat you would simply look up the price in the newspaper. This heterogeneity gives rise to a distribution of possible prices.

An international accounting standards definition from the International Association of Assessing Officers (IAAO) and adopted by RICS and the American Appraisal Institute reads:

“The most probable price which a property should bring in a competitive and open market under all conditions requisite to a fair sale, the buyer and seller each acting prudently and knowledgeably, and assuming the price is not affected by undue stimulus. Implicit in the definition is the consummation of a sale as of a specified date and the passing of title from seller to buyer under conditions whereby:

- 1) Buyer and seller are typically motivated;
- 2) Both parties are well informed or well advised, and acting in what they consider their own best interests;
- 3) A reasonable time is allowed for exposure in the open market;

- 4) Payment is made in terms of cash in U.S. dollars or in terms of financial arrangements comparable thereto;
- 5) The price represents the normal consideration for the property sold unaffected by special or creative financing or sale concessions granted by anyone associated with the sale.” (Appraisal Institute, 1999:5)

The International Valuation Standards Committee (IVSC) definition is more succinct and is recommended by the Australian Property Institute:

“Market value is the estimated amount for which an asset should exchange on the date of valuation between a willing buyer and a willing seller in an arm’s-length transaction after proper marketing wherein the parties had each acted knowledgeably, prudently and without compulsion.” (Australian Property Institute, 1999:44)

The definition can be made even simpler: An American real estate website gives an example of a clothing item normally selling for \$25 but later marked down to \$10 due to lack of demand. Echoing Alfred Marshall, the website concludes that “market value is simply the price at which something will sell within a reasonable period of time.” And therefore proposes as a definition of value:

“Market value is the price at which a particular house in its current condition, will sell within 30 to 90 days.” (http://biz.yahoo.com/edu/re/ir_re5.ir.html)

Definitions that go in this direction—that is, leaving out the various clauses about informed buyers and so on, may be clearer than the more elaborate definitions. The implicit assumption is simply that the circumstances of sales are what they are and that moreover, the valuer knows what they are and takes them into account. The appraiser is charged to look at actual market conditions, not hypothetical circumstances. After all, actual conditions will determine price.

A State of Massachusetts continuing education course for brokers says

“An appraisal is an unbiased estimate of the nature, quality, value or utility of an interest in, or aspect of, identified real estate. . . . Valuation is the process of estimating market value, investment value, insurable value, or other properly defined value of an

identified interest in a specific parcel at a given date.”

(<http://www.state.ma.us/reg/boards/re/contedu/2001/026.htm>)

This definition shows that the valuer has to get right the purpose of the valuation to find the relevant value definition and moreover, to understand correctly the legal rights being valued.¹

The Appraisal Foundation USPAP2002 definition, the most up to date of those reported here continues the trend towards asking valuers to look at the actual purpose and circumstances of the price estimate and adapt their methods and value concept accordingly:

“MARKET VALUE: a type of value, stated as an opinion, that presumes the transfer of a property (i.e., a right of ownership or a bundle of such rights), as of a certain date, under specific conditions set forth in the definition of the term identified by the appraiser as applicable in an appraisal.

Comment: Forming an opinion of market value is the purpose of many real property appraisal assignments, particularly when the client’s intended use includes more than one intended user. The conditions included in market value definitions establish market perspectives for development of the opinion. These conditions may vary from definition to definition but generally fall into three categories:

1. the relationship, knowledge, and motivation of the parties (i.e., seller and buyer);
2. the terms of sale (e.g., cash, cash equivalent, or other terms); and
3. the conditions of sale (e.g., exposure in a competitive market for a reasonable time prior to sale).

Appraisers are cautioned to identify the exact definition of market value, and its authority, applicable in each appraisal completed for the purpose of market value.”

(<http://www.appraisalfoundation.org/html/USPAP2002/DEFINITIONS.htm>)

Note that this definition finally allows valuers the freedom to look at actual market conditions, suggesting the checklist of issues to describe in characterising the transaction.

Academic critics of court and professional society definitions

Academic authors have been critical of court-mandated definitions based on an artificial set of conditions or assumptions however stated. Whipple cites articles by Halbert Smith (1977, 1986) and Albritton (1980) where the various words in these

definitions are examined and found to be vague, open to interpretation and possibly different from the actual circumstances of particular sales. Whipple, therefore, points out that these definitions that state particular assumptions about buyers knowledge, circumstances of sale, etc. are “normative” that is they are definitions that specify how things “should be” rather than how things actually are.

If the actual circumstances differ from those in the normative definition, then the price that will occur in a market would also be likely to differ—that is, the valuation would give a misleading or wrong answer. An egregious example was when Alan Bond (later jailed for looting companies) instructed a valuer to value an office building site “as if completed and fully tenanted at prevailing market rents” or words to that effect. The resulting valuation would best be classified as part of the crime fiction or perhaps fantasy genres rather than information to be relied upon by investors. The valuer’s defence was that nobody asked about the likelihood of actually being able to find tenants in a market with severe oversupply conditions or the likelihood that rents would fall under such conditions. This is an extreme example, but logically similar to cases where we assume knowledgeable buyers in a market actually consisting of naïve optimists. Valuers aiming at the wrong target are unlikely to hit the right one.

The IVSC states in its “General Valuation Concepts and Principles” that “Imprecision of language, particularly in an international community, can and does lead to misinterpretations and misunderstandings...Importantly this is the case with the terms price, cost, market and value as they are used in the valuation discipline.” (Appraisal Institute, 1999:8)

Professor Ratcliff, a former estate agent, wrote that in most cases, clients simply want to know the most likely sale price in the market as it actually exists. They are not interested in a fictitious market of rational, well-informed typically motivated fully informed buyers and sellers if such assumptions lead to a misleading estimate of a property’s likely selling price. As Ratcliff put it "appraisers would not be hired unless clients were faced with some real estate decision or problem"...the client’s needs will define the nature of the value figure which the appraiser is asked to derive. And having identified the value figure which is to be sought, we shall have made the first

step toward understanding the analytical process which will generate this value figure." (Ratcliff, 1972:1) Ratcliff concludes "Market value or the probable selling price of the property is of primary concern" to most clients. (Ratcliff, 1972: 4)

Whipple calls this kind of value definition a "positive" definition, in that it postulates no conditions except those actually observed in the market itself. The term "positive" here is used in its philosophy of science meaning, associated with "positivist empiricism" where results rely on empirical observation. Positive economics is concerned with *what is*, normative economics with what *should be*, regardless of actual conditions.

In his valuation textbook *Property Valuation and Analysis*, Whipple cites U.S. Supreme Court Justice Brandeis who said "value is a word of many meanings." Whipple cites authorities who offer differing numbers of value definitions for various purposes: Westwick (9 kinds of value), Ring (30 value concepts, McMichael (54 varieties). (Whipple, 1995:76) API professional practice standards outline in some detail several alternatives to the IVSC market value definition including going concern values, forced sale values, due diligence values and even Native Title values. The API notes make clear that definitions and methods may need to be adapted to the client's problem and methods adjusted accordingly to avoid biased or misleading estimates. (Australian Property Institute, 1999)

The Appraisal Foundation's white paper on market value states that "Clients have needs for varying types of values and real estate related information depending on the business decision involved and level of risk. Such values may include, but are not limited to, investment value; use value; recommended listing price; reasonable selling price; pricing opinion; liquidation value, anticipated sales price, and fair price. . . . "Methods and procedures designed to develop and report Market Value (see above definition) may be different than methods and procedures appropriate for developing and reporting values other than Market Value." (Appraisal Institute, 1999:8).

Whipple insists that the appropriate definition of value depends on the purpose of the valuation. For example, statutes and court decisions may define value in a resumption case, while replacement cost may be relevant in an insurance valuation. He points out that "normative" definitions, such as the Spencer Case and other "stated assumptions"

definitions like those cited above, create artificial situations that may not conform to the actual conditions of a sale. In this case, the value definition invites the valuer to come to a mistaken or biased price estimate.

According to Whipple, valuations must follow a set of logical steps that begin with identifying a purpose of the valuation. Only then can an appropriate definition of value be chosen from among many possible candidates. Usually, as Ratcliff pointed out, the idea is to predict or estimate what a property would sell for under actual conditions, so a “probable” price definition *a la* Ratcliff is often the best choice. An implication is that the valuer needs to describe his assumptions about market conditions in some detail so that the client who relies on the valuation understands clearly the value definition used and can follow the evidence and reasoning used in arriving at a probable price estimate and the uncertainty inherent in the estimate.

A statistical value definition

The proposed statistical definition of value has four clauses:

- Estimates of parameters of the subject property’s possible sale price distribution
- Estimates of errors in the parameter estimates and diagnostic tests.
- Forecasts of the stability of the estimates over a relevant period.
- Statements of explicit assumptions about the circumstances of sale including legal rights valued, date of sale, method of sale, time on the market, finance, probable uses of the property valued, probable buyers and motives/knowledge of buyers and sellers.

The value definition necessarily incorporates by assumption the valuer’s methods and data as an implicit part of the definition. Methods and definition are inseparable and rely on one another. The parameter estimates mentioned in the definition are only meaningful if they emerge from proper methods and data.

This definition assumes market prices are revealed by market transactions, but that valuations can benefit by taking into account deviations from assumptions required for market efficiency in the subject case. Traditional definitions (prior to the USPAP2002 version) require the valuer who takes them literally to assume market

efficiency. This statistical definition, like the 2002 USPAP definition, requires the valuer to report the price estimate based on the market as it is observed to be in its current state, efficient or not.

Statistical definition as a clarification of Ratcliff's definition

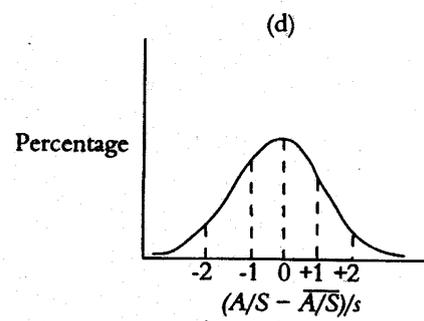
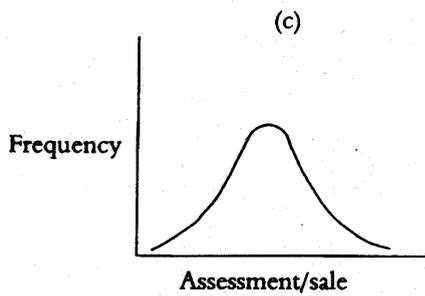
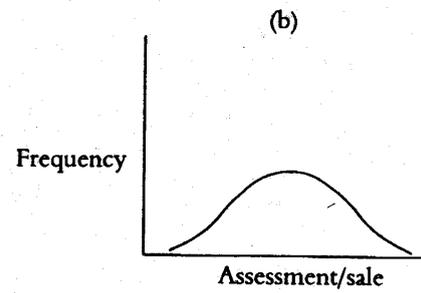
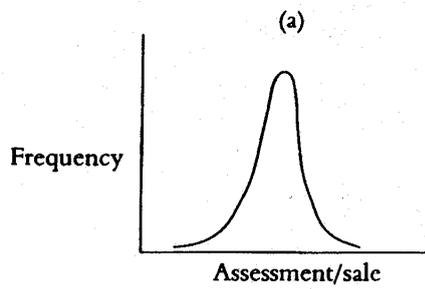
The statistical definition of value given above can be thought of as a clarification or expansion of Ratcliff's notion of "most probable price" that would emerge from a market transaction. Ratcliff's definition of "most probable price" is open to various interpretations. Does he mean the median, mode, mean, highest possible bid, most likely bid or what? (Colwell, 1979) Ratcliff advocated that valuers report value as a range rather than a point estimate and developed an argument for a "transaction zone," that is a range created by the overlap between the lowest offer a seller might accept being less than the highest offer a buyer might make. In this range, the price is set by negotiation.

Using the statistical definition cited above clarifies the central tendency and range ideas of Ratcliff's probable price concept: To describe a distribution, we can choose any set of the summary statistics. All are well defined and widely understood. Each imparts different information so looking at several summary statistics would often be helpful. For example, because price distributions are often skewed and the median is more robust to outliers than the mean, probably median of the possible price distribution would be the preferred measure of central tendency. A 90% confidence interval might be an easily interpretable measure of variability.

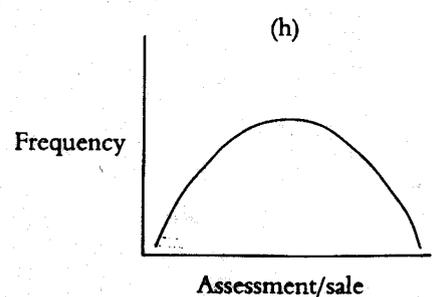
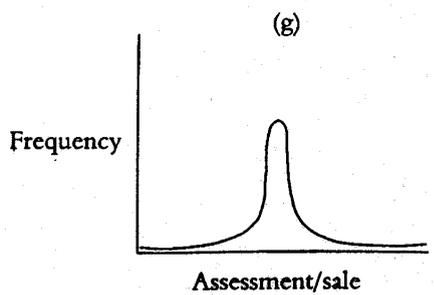
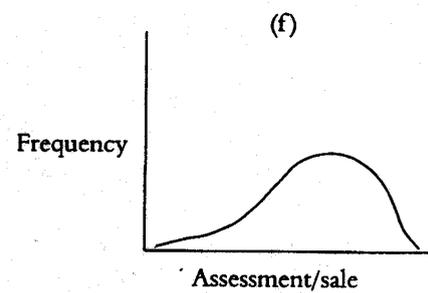
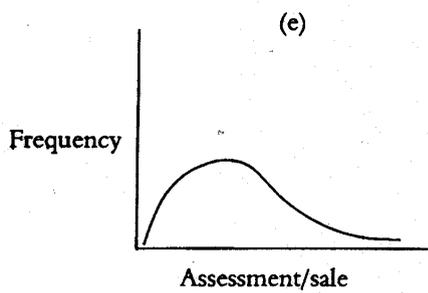
This definition, if accepted, requires valuers to be familiar with basic statistical concepts like distributions, probability and descriptive statistics and therefore has implications for real estate educators. However, the statistics involved are so simple that they can be quickly learned from any introductory textbook, introductory statistics website or similar source. No valuer should leave home without at least an elementary understanding of statistical concepts in any case. Valuers need to use summary statistics daily in deriving price estimates from diverse data. Examples of distributions from IAAO's textbook appear in Figure 1. A distribution is simply a function relating the values of a variable (in this case possible price) to the relative frequency of each possible outcome.

Figure 1 Examples of data distributions.

Normal distributions:



Nonnormal distributions:



Source: IAAO, 1999:94

There are several distributions relevant in valuations:

- Distribution of sales in a geographical area over a period of time.
- Distribution of possible valuations of a particular subject property.
- Distribution of possible sale prices of a particular subject property as of a given date.

The first is the pool of potential comparable sales, a distribution representing all sales in a market or submarket. The second distribution reflects the variation in price estimates when more than one valuer estimates value, reflecting uncertainty and errors in valuation methods. The third is the “target,” the relevant definition of value that a valuation should report.

The statistical definition has several virtues in comparison to traditional definitions:

- It connects valuation to a large body of statistical method and technique and therefore gives us a language for talking about values more coherently. Because the mathematical definitions of concepts like distributions, means, modes, standard deviations, range, skewness, outliers, etc. are relatively clear and precise, this language avoids some of the ambiguity present in traditional verbal definitions of value.
- A statistical definition focuses attention on variability and hence on risk. Therefore it adds significant useful information for clients in comparison to traditional language. Traditional valuation methods invite unsophisticated clients to make the mistake of regarding value as something intrinsic to a property that the valuer has been able to measure—like ceiling heights or square meters of floor space—rather than the ever changing outcome of two blades of a supply/demand scissors operating in a market.
- Looking at value as a distribution puts valuation methods on sounder theoretical footing and helps clear up confusion about methods.
- Admitting that price is a distribution appears to offer some comfort to valuers plagued by professional negligence claims from dissatisfied clients or stakeholders. If possible price is a distribution, then a wide variety of outcomes are in fact possible, so the focus in negligence cases would properly shift to methods rather than outcomes.

The statistical approach to value definitions has some disadvantages as well.

- It leaves valuers more scope for variation in their discussions of the circumstances of sale that are so important in traditional definitions. Less standardisation of assumptions might leave latitude for abuses. The fourth clause in my definition above says the valuer has to do a proper valuation for the estimates to be meaningful. It is questionable to define value without incorporating by reference all the issues in a proper valuation process necessary to arrive at an objective estimate incorporating a reasonably complete set of information that determines price. If market prices imply some kind of information efficiency or at least some set of information, efficient or not, then clearly the information content of the valuation will influence the value estimate. The value definition has to reference the valuation method to ensure complete, competent, objective information goes into the value estimate. Indeed, the distribution of possible prices will reflect the information the market is using to arrive at transaction prices. The definition becomes moot if the valuation method is biased or incomplete.
- It would require decades for an alternative definition to work its way into court precedents. For the immediate future at least, the inertia of the law will require that valuers at least pay lip service to traditional, court sanctioned definitions. Valuation standards and professional societies also have in place preferred definitions of value that may have to be adhered to by practitioners and that will change slowly. Professional society standards cannot lead court sanctioned definitions by much. Therefore I propose the statistical definition as a means of clarifying a valuer's own thinking and methods, and a supplement to, rather than as a replacement for traditional verbal definitions.
- The statistical definition requires some re-education of valuers and modification of valuation methods.
- Probably the loudest complaints would come from traditional valuers with respect to the third clause in the proposed definition—the one requiring forward looking forecasts. Valuers have seen their role as analysing current market conditions, not as forecasting. This clause in the proposed definition, therefore, asks valuers to take on a new role. I think they should do so, and happily, because it is an area where their expertise could add value for clients and hence enhance the market for valuers' services. Moreover, market cycles,

theoretically, come from refusal of markets to look forward. Market cycles that are so costly to investors and the industry might be mitigated if experts were instructed to look forwards to forecast impending shortages or oversupply. Another reason why valuers should take on the forecasting task is simply that according to universally accepted finance theory, current values depend upon future benefits of ownership. Finance theory should compel forward-looking valuations. Current markets may misprice and for valuers to perpetuate or magnify mispricing by relying only on recent sales abrogates a considerable positive role they could play in the real estate process. Of course, this assumes that the profession has some integrity and expertise. If it does not, then forecasting would open new worlds of client abuse and professional indemnity.

Tradition is no reason to maintain a definition if the definition is problematic or leads to valuation errors. The statistical definition should add value for clients and result in less biased, more replicable, defensible, logical and accurate valuations.

Value definitions such as the Australian Spencer case arose from court cases where there were involuntary takings of property. The courts sought to describe the circumstances of a typical sale, where the price would be fair to the landowner and the public agency acquiring the land. Academic authors have been highly critical of court mandated definitions based on an artificial set of “normative” conditions or assumptions that could lead to artificial value conclusions if they are applied to actual markets whose circumstances differ from the standard.

The resolution is to choose the academic’s “positivist” camp, that is, to regard appraisal as a science interested in the real world as it actually is, while recognising that the traditional definitions provide a checklist of important issues valuers need to observe as they make their estimates of possible sale price distributions under the definition appropriate to the client’s problem. This is where the professions are headed, as in the USPAP 2002 definition cited above.

Valuation defined as estimation of summary statistics of an unobservable distribution of possible prices

Summary statistics of a distribution of outcomes (or possible outcomes in the valuation case) include:

- Measures of central tendency: mean, mode, median
- Measures of dispersion or variation: Variance, standard deviation, range.
- Skewness
- Kurtosis (flatness vs peakedness of the distribution)
- Analysis of outliers: Are there unusual points and why?

Graphing the relative frequency of different outcomes may reveal additional detail about a distribution—histograms, box and whisker plots, etc. can reveal patterns in price data. All of these summary measures have unambiguous definitions and can be calculated for any population of actual sales.

The tricky part of the statistical value definition is that one cannot observe more than one point from a distribution of possible sales. We only get to see the one sale that actually happens on the day. We have to infer or imagine the possibility that other prices might have occurred based on the insight that the observed price is an outcome from a random process in a heterogeneous market. There could have been other prices besides the observed price. Valuers do not know the “true price” (a meaningless concept) and neither do buyers and sellers. There is a transaction price that eventuates under particular unique historical circumstances.

Academic literature encompassing hundreds of articles implicitly defines value as a random variable. Every hedonic regression equation includes an error term, $\text{price} = \text{xb} + e$ where e is a random error. Any model ever proposed, has been incomplete—additional unmeasured sources of price variation omitted from the model lead to random price variation around the most estimated price. Therefore, clearly, academic authors customarily treat price as a random variable, that is, price has a distribution or variability described by an error term. Colwell remarked that “There is a distribution of potential selling prices for any subject property to be appraised.” (Colwell, 1979:106)

In ordinary least squares estimation (OLS), errors should be normally distributed. The price estimate \hat{p} is the central tendency or most probable estimate from a distribution of possible prices with variance estimated through the equation's error term. Pricing models are not deterministic, but probabilistic. If OLS assumptions are met, we can calculate a confidence interval for the price estimate. But it is not necessary for present purposes that price errors be normally distributed—any distribution of errors means price shows variability and therefore should be thought of as a distribution.

For the statistically minded, the preceding paragraph should be enough to make the case that price is a random variable and therefore a sensible value definition can be “estimated summary statistics of the possible price distribution.” Others may need a more qualitative discussion to prove to their satisfaction that price is indeed a random variable with outcomes a distribution of possible prices.

Let's start on Sunday morning. You've gone shopping for a house, while I've slept in. You like the kitchen more than I do so you bid \$5000 more for the house than I would have. The seller has had it on the market for months and would have taken either of our offers. So if you had slept in instead of me, the house might have sold for \$5000 less. This is how we experience random price variation every day. People say things like “he got a good price” or “it sold cheap,” suggesting they have in mind a distribution of possible prices a property might have sold for.

To me the conclusive emotional test as to whether price is a single deterministic dollar figure versus a hard-to-estimate distribution comes in the experience of buying property. My experience has been, while agonising about whether or not to accept an offer or to make a higher offer in response to a counteroffer, that there is no way to know the exact value of a property. Sometimes you bid the extra dollars and sometimes you don't, but you are never sure you've got it right and a case could always be made for paying more or less. Buyer uncertainty reflects the complexity of the decision problem where the future benefits of ownership depend on many factors, some known, some unknown, some changing over time. No one has the complete information called for by traditional verbal definitions of value. Differing incomplete information sets lead to differing transaction prices.

There is plenty of additional “common sense” market evidence that a range of prices is possible for a particular property on a particular day that give clues to the nature of the unobserved possible price distribution. When properties are offered for sale, the “listing price” at which they are offered, most often (but not always) exceeds the final selling price. Sellers list properties at prices slightly above what they expect to achieve to reflect their own and the market’s uncertainty about the “correct” price. Some agents reflect this uncertainty by advertising price ranges “offers accepted between \$180,000 and \$220,000,” for example. There is no single “correct” price, but only the imperfectly estimated summary statistics (mean, standard deviation, skewness, range, etc.) of a possible sale price distribution. That is the best the market can do and valuers can do no better.

Reasons for price variation, in addition to the buyer characteristics (or more formally, differences in the buyer’s hedonic pricing model or demand function), include different information sets used by different buyers. You have heard the school will be upgraded, I haven’t, or I notice that the roof leaks but you don’t. In addition, circumstances of sale may vary. One seller may be more motivated than another for any number of reasons. Interest rates available to different buyers may vary. And so on. Real estate, as it says in chapter one of our textbooks, is heterogeneous, and this applies to characteristics of properties, buyers, sellers and circumstances of transactions.

Negotiation or market power is also important. Typically, as discussed by Ratcliff, there is a “transaction zone” wherein the price is above the lowest offer the seller might accept and below the highest offer the buyer might make. Where exactly the sale price occurs in this zone of overlap has to do with negotiation.

Ratcliff emphasised, therefore, that a valuation is a prediction of human behaviour under uncertainty. He recommended trying to understand how “probable buyers” would think in determining their bids for a property. Human behaviour tends to be somewhat unpredictable or variable or looked at another way, to be a product of exceedingly complex cognitive processes and experience. This leads to variation in prices that might be paid. (Ratcliff, 1972)

According to finance theory established since the 1920s, the value of assets depends upon future benefits of ownership. But the future is inherently uncertain (See Popper's argument against Marxist Historical Materialism, Popper, 1974), therefore, present value is uncertain in every case. All asset values are random variables. Price is based on uncertain and risky estimates of future benefits. So price must include risk and uncertainty as well.

Adding option pricing terms to pricing models may improve estimates but does not get rid of potential variability in pricing because several variables in the option pricing model are measured with uncertainty or are uncertain future events. *When Genius Failed* about Long Term Capital Management's loss of \$4 billion U.S. dollars describes the failure of Nobel Prize winners in finance to correctly predict future variance. Option prices are random variables too. (Lowenstein, 2000)

Much of the variability of possible prices has to do with heterogeneity and infrequent trading in property markets. People simply do not know very precisely what a house is "worth" that is, the price it will sell for. There is "no market price in real estate, only deals" as the head of real estate for Merrill Lynch remarked. Indeed, uncertainty about what price may be achieved is the reason for the valuation profession in the first place. If prices were precisely known, there would be little need for valuations. Valuers make price estimates and no one should be surprised when valuations and sales prices disagree or when valuers disagree with each other.

Implications for valuation methods and products

Valuation methods

If price is a random variable, then the observed price in any particular sale is an observation drawn from a distribution of possible prices. This means prices do not always occur at the most likely price. In fact, sales usually would not take place at the most probable price, the mean or median of the distribution of possible prices.

Therefore, observed prices are evidence, but not conclusive evidence. Estimates of a property's value based on a single comparable sale could be biased by the random variation in the observed sale price of the comparable. It might have sold "high" or "low" in comparison to its most likely sale price. This buyer "error" in the

comparable sale price would then flow through to an estimate of the subject's price based on that comparable sale. (Kummerow, 2000)

Consider some of the implications for valuation methods:

Any single sale price could be misleading. Any small sample of sales could be misleading due to random errors. As sample size increases, the law of large numbers says mean errors will sum to closer to zero, that is the mean sample estimate will approach the population parameter. This means getting an adequate sized sample to "average away" random errors is important.

However, bigger samples are not always better because the heterogeneity of properties means that bigger samples will be less similar. So estimates of price can get less precise as sample size increases, if there are measurement errors, errors in the model used to value the property and adjust for differences or if population variance increases with population size. The implications are that one needs to search for a sample that is big enough to give a stable estimate of price, but small enough to avoid the misspecification biases and other problems that can occur with larger samples. (Kummerow & Galfalvy, 2002)

Two identical, mirror image halves a duplex are for sale. One sells for \$310,000. What will the other sell for? Well, probably for around \$310,000 because markets tend to be fairly efficient. But perhaps not. So, rather than accepting the \$310,000 sale as conclusive, even for an exactly identical property, a valuer should seek additional confirming sales evidence. If the mean of 5 adjusted sales was \$305,000, that might be a better estimate. The best sample size depends on the relative magnitude of the random pricing errors made by buyers versus adjustment errors made by the valuer. See Kummerow & Galfalvy, 2002 for a discussion of these error tradeoffs.

Paired sales are recommended in valuation textbooks as a way to determine the value implications of a particular property feature. But any paired sales analysis based on a single pair of sales or even a small sample of sales, could be misleading due to random variation in the selling prices. Say two properties are identical except that one has a tennis court and the other does not. If the one with a tennis court sold for \$10,000 more than the one without the tennis court, does that mean a tennis court is

worth \$10,000? Definitely not. Perhaps the one with a court sold below the mean of its possible price distribution while the other house sold above the mean of its possible price distribution. In that case, the paired sales difference in price would understate the value of the tennis court. And vice versa if the house with the tennis court sold for more than the mean of its possible price distribution.

The purpose of this paper is simply to propose the need for focussing on the unobservable possible price distribution concept. Space does not permit discussion of methods for estimating the distribution's parameters. Kummerow, 2000, and Kummerow and Galfalvy, 2002 provide some initial thoughts on these difficult problems.

But suffice it to say here that everyone involved in markets has some idea of the parameters of the possible price distributions. Value estimates produced by valuers are estimates of the mean or median of the unobserved distribution. Most valuers, if pressed, could probably produce an estimate of the range of their value estimate or a 90% confidence interval. Kummerow, 2000 suggests 7 "clues" to the variability of the unobservable possible price distribution. This is clearly an area for further research to identify practical standard methods.

The variation in price estimates derived by adjusting different comparable sales provides an indication of the uncertainty of the valuation figure, although this variation may be partly due to adjustment errors by the valuer as well. However, if the indicated values from three comparables are all within \$5000 of each other, the valuer is justified in concluding that the variance of the possible price distribution for the subject is probably less than if the three comparables were scattered across a range of, say \$25,000. If a number of appraisers were asked their opinion of the value of a property, the distribution of the valuations might give another reasonable clue to the nature of the possible price distribution.

The possible price distribution might be revealed by the range of prices for a batch of similar lots sold at auction on a given day under similar terms. Or by the distribution of differences between listing prices and selling prices in a particular neighbourhood or property type.

Prediction error statistics may say as much about the variation of prices as they do about the inaccuracy of valuations. Probably the key statistics for valuers to focus on are their own valuation errors—what are the differences between valuations and subsequent sale prices. In asking valuers to define value as the parameters of the distribution of possible sale prices, we can check whether they do so by looking at statistics about the errors in valuations. These prediction errors are the issue of interest to valuers and clients.

In the end, whether the errors are due to the variation in the sale price itself or the errors in the valuation process, we are interested in matching the valuation estimates to the market outcomes. Valuers should not claim more accuracy than they are capable of delivering. Using a statistical value definition that builds in estimates of errors gives valuers a simple way to report the uncertainty in markets and their own reading of markets.

Valuation products

The proposed statistical value definition suggests the following generic valuation products:

- 1) The central tendency estimates of a traditional valuation, but more precisely defined as summary statistics of a possible price distribution. This, and supporting evidence is all that the traditional definition requires to be reported. It is a product of diminishing value in a rapidly changing world of risk and uncertainty. The credibility of the “point estimate” has been damaged by studies and plaintiffs’ actions showing that claims of accuracy are spurious.
- 2) The distribution “spread” measures reported through the variability parameter estimates. This is a product that might be called a “risk and uncertainty study.” It will add value wherever investment decisions depend on assessment of possible variation in returns, i.e. in all investment decisions.
- 3) The forecasts provide both a picture of the risk of the investment and valuable information for investment strategies based on rational expectations. The size and revenues of the forecasting industry are large and this is an area valuers should seek to enter.

- 4) Deeper insight into the issues affecting value and the correct way to maximise returns could come from the descriptions of market circumstances including submarket analysis, tenant preferences, state of information of buyers and sellers, hedonic pricing models and all the other aspects of the valuer's knowledge and expertise inherent in the methods described in a full report.

So now we envisage a valuation profession able and willing to offer a complete range of property advice in response to the client needs mentioned in the Appraisal Institute 1999 White Paper cited above.

Implications for clients and the profession

A statistical definition's main advantage over traditional definitions comes through the implication that additional summary statistics need to be reported beyond simply the mean or expectation of the possible price distribution.

The second moment of the distribution, the estimated variance, provides clients with important information about risk. It also provides valuers with important protection against unjustified negligence claims. There are three possible explanations for variation between valuations and the later realised sale prices of properties.

- First, the valuer may be negligent—there may be bias or faulty reasoning in the valuation. There may be flaws in method, data or interpretation.
- Second, the observed sale may be a low probability outcome—the occasional improbable low or high sale that a distribution of possible sales could produce as a single outcome. The improbable, as a popular bumper sticker says more graphically, happens.
- Third, markets change so the possible sales price distribution itself can vary through time. Price indexes or other measures of price change may give some idea of this, but valuers should not be blamed for bad outcomes due to unforeseeable changes in market prices over time.

In the latter two eventualities, the valuer is off the hook. I propose to put him/her back on the hook with respect to forecasts by adding a projection of future market conditions to the valuation. This would, as I have said above, add value for clients

and could improve market efficiency. And it is difficult for me to understand how if finance theory tells us values depend on future conditions we can ignore future conditions in making value estimates.

The third moment of the possible price distribution, skewness, gives an idea about whether to expect symmetric results versus a tendency for larger errors in one direction or another. Risks often are not symmetrical and clients would be very interested to have estimates about upside and downside asymmetries.

Statistical definitions can be more precise than verbal definitions. Instead of saying “well-informed buyers” and then wondering what that means, simply cut off outliers above or below what you estimate to be three standard deviations from the mean. Tell the client what you have done. This provides a clear and replicable method for operationalising the concept of “well-informed” or “not compelled to sell.” The mathematical definition leads towards a replicable valuation method rather than an unresolvable argument between possibly biased opposing opinions based on assessments of buyers’ mental states of informedness and motivation.

Summary

Traditional methods of defining value suffer from vagueness and lack of replicability. They may be part of the reason for the embarrassing (to the profession) frequency with which valuers on opposing sides of court cases testify to dramatically different values.

Defining value as a random variable to be measured through empirical evidence leads towards clearer thinking and more replicable and less biased valuation methods. Algorithms can be developed to take us from a given set of sales evidence to a value conclusion in a way that any other valuer could replicate. The statistical definition also provides a rationale for thinking about sample size—that is, how many comparable sales to use in a valuation. It also helps prevent errors such as relying on a single sale price or a single set of paired sales that could be misleading due to random variation.

Defining value as the parameters of an unobservable distribution of possible prices raises some difficult issues regarding how to estimate these parameters. I attempt to explain some clues to these parameters in a 2000 *Appraisal Journal* article (Kummerow, 2000). That article is by no means the last word on the subject. Much more can be done by looking at prediction errors and how those depend on sample sizes. Because it is hard to estimate something precisely doesn't mean we can't get an unbiased imprecise estimate that is useful. Although the possible price distribution is unobservable, the combination of pricing errors and valuation errors shows up clearly in the *ex post* distribution of valuation price estimates versus actual sale prices. This "combined" buyer and valuer error is the relevant error when valuations are evaluated, so statistics on valuation errors can provide useful means of estimating the variability of the possible price distribution (grossed up by addition of valuation errors). Valuers should be tracking valuation errors as a quality control measure and can incorporate results in their valuation reports.

The mathematical definition adds value in two important ways. First, it provides the valuer with a defence and a rationale for cases when subsequent sales prices do not equal valuation estimates. The statistical answer is: There is random variation in sales prices so it would be unlikely for sales prices to equal valuations. Even large deviations are statistically possible—all distributions are subject to "outliers," unusual events that occur by chance or due to some unexplained factor. So the definition adds value for valuers in providing them with a valid defence against professional negligence claims in cases where negligence has not occurred in the methods used to arrive at estimates of the possible price distribution parameters. Of course, it also provides a noose for the neck of valuers who systematically bias results or who do not follow methods that can be justified based on the evidence.

The statistical definition suggests statistical methods that can make an objective case for particular value estimates. Even valuers who contend valuation is an art form rather than a science (i.e. who use their "gut feel" and qualitative factors based on experience to reach conclusions and interpret market evidence) can check their work by looking at error distributions. They may very well produce better valuations than those using hedonic regression or adjustment grid methods. But they should still

report the uncertainty of their estimates and the possible variation in the values they propose.

On the client side, the definition adds value by bringing risk and uncertainty explicitly into the valuation product. Most clients would be quite interested to know the range of possible outcomes and the probability of outcomes below or above particular values. This is not a world of certainty but rather a world of risk management. Valuations that report the second and third moments of price distributions in addition to the measures of central tendency must certainly be more valuable to clients for many purposes. In some cases, such as valuing a house for a divorce settlement, all that is needed is a single number for the Family Court to ratify and use as the basis for splitting the marital assets. But in cases where money is at risk—investments, lending decisions, lease negotiations, etc. it would be useful to clients to know how likely different outcomes are. At times, knowing the risk would change the decision.

Even more valuable would be some idea of the future direction of market values. Forecasts should become a standard part of a value definition and a valuation report—adding a time dimension or graph of value over time would add value. Market inefficiency and market cycles—if Lucas is right about rational expectations—must surely be partly due to lack of forward looking valuation methods. Forecasts should eliminate regular, predictable cycles.

Price as a random variable has been a foundation assumption for academic research methods on real estate prices. It is a theoretically sound platform, built on fundamental Economics and Finance theory, upon which to build valuation research, valuation practice and client decisions based on valuation results. Once the unfamiliarity of thinking about parameters of unobservable distributions wears off, practitioners will find it easier to do valuations based on the random variable concept rather than the more confusing and contentious verbal definitions.

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¹ The Appraisal Foundation, a combination of eight U.S. valuation societies reports that "the most commonly utilized market value definition for business enterprises is found in I.R.S. Revenue Ruling 59-60, which state:"The price at which the property would change hands between a willing buyer and a willing seller when the former is not under any compulsion to buy and the latter is not under any compulsion to sell, both parties having reasonable knowledge of relevant facts. Court decisions frequently state in addition that the hypothetical buyer and seller are assumed to be able, as well as willing, to trade and to be well informed about the property and concerning the market for such property." (Business valuation Market Value Definition - USPAP Standard 10-2(c))