VALUE OF COLLECTIVE EFFORT: REDEVELOPMENT TIMING OPTIONS OF ENBLOC SALE SITES IN SINGAPORE

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ABSTRACT

This paper applies the discrete-time binomial tree option-pricing framework proposed by Cox, Ross and Rubinstein (1979) to evaluate redevelopment options embedded in collective sale sites in Singapore. Based on 30 collective sale transactions recorded in 1999, our empirical results showed that the average redevelopment option premium was estimated at S$12.24 million or 9.65% of the gross development value. By adding the redevelopment-timing premium to the developer's profit and returns of 15%, the developer would expect a premium of 24.65% of gross development value if he were to exercise his option to redevelop now. Otherwise, it will be more valuable to keep the redevelopment option "alive." The regression results further showed that the option premiums are positively and significantly related to the size and the price of the collective sale sites. Sensitivity analyses were also conducted and the results showed that the redevelopment option premiums increase when the price volatility, which is measured by the spread between the upward (u) and downward (d) price movements, increases. Changes in risk-free interest rates have positive effects on the option premiums.

Keywords: Binomial tree option pricing model, collective sale, redevelopment
option.

INTRODUCTION

The millionaires of Changi Heights...

> *"A group of 58 property owners in Changi have reaped a windfall of (S)$2 million apiece by collectively selling their entire housing estate to a private developer. The owners of the 30-year-old Changi Heights estate will together collect a cool (S)$116 million from residential developer Ban Hin Leong..."*
> (Abdul Hadhi, Business Times, 2 November 1994)

Collective, or en-bloc sale has silver-lined the residential property market of Singapore following the successful sale of Cosy Mansion in October 1994. The 20 owners of Cosy Mansion reaped substantial financial windfalls from the transaction. 58 owners of another freehold condominium - Changi Heights, also realized windfall gains by selling en-bloc the development in November 1994. The boom of the Singapore residential market in 1994, and the revision of the development guide plans[1] by the Urban Redevelopment Authority[2] that raises the density of many residential development sites have been the contributing factors for the surge in collective sale activities in 1994 and 1995.

En-bloc sales involve collective actions of owners of more than one private residential property in either a multi-tenanted development or contiguous landed developments. They come together to amalgamate their joint interests in the properties and cash-in potential financial gains by selling the redevelopment rights of the land. The windfall, which motivates individual owners to participate in the en-bloc sales, is created when the potential value of the land surpasses the *"marriage"* value of the land[3] and the physical improvement thereon. The upward revision of land use potential of a collective sale site under the revised development guide plans, the ageing of the physical building thereon, and the depreciation of its market value are economic factors underpinning the "windfall accumulation" process (Low, 1999).

The windfall profits were generated in many collective sales by transferring the redevelopment rights at a value that is higher than the *"marriage"* value of the land and the existing structure. Instead of sitting on land with latent development value, the rational decision is for the owners to exchange the development rights attached to the land for an economic gain with developers, who have the ability to exploit the redevelopment potential of the land. In those en-bloc sites that have been transacted, windfall gains were lucrative (Lum, Sim and Lee, 1999). Cosy Mansion's owners, for instance, realized an average windfall profit of S$700,000 each, whereas the owners in the Changi Heights's collective sale have made an estimated gain of S$800,000 to S$1,300,000. In another S$73.6 million deal involving five bungalows off Walshe Road, the owners doubled their value with a gross proceed of $14.7 million each by banding together in the sale instead of selling the bungalows separately (Abdul Hadhi, 1994b).

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[1] Development guide plans are micro-level plans that lay out the development guidelines, which include the height control, zoning and density of development, for the scarce land resources in Singapore.

[2] The Urban Redevelopment Authority (URA) is the national planning authority of Singapore that is responsible for the long-term land-use planning and development control in Singapore.

[3] "Marriage" value arises when the sum of parts is less than the whole. It usually exists in a land assembled from several smaller parcels of contiguous lands. The value of the merged land is higher than the sum of the individual parcels of land valued separately.

However, windfall profits are not always the sole motivation that entices owners to part the ownerships of their properties. In some cases, owners have refused to follow the en-bloc bandwagon because of the intangible values they have attached to the quality of life and the environment they have been familiar with. The sentiments of these owners are reflected in the two newspaper extracts below:

"... I know there is profit involved, but I don't want the profit. I want a home and this is the home I have made for myself – surely it has to count for something..." (Tan, 1999b)

"... there are others who don't want to sell because they are quite happy to continue staying in a neighbourhood they've come to perceive as "home". No doubt, an en bloc's gain involved many individuals financial profit, from the maximization of land under an increased plot ratio. But an en bloc's loss is something unquantifiable – the loss of a home. And, in some cases at least, a neighbourhood's original serenity, its large open areas and mature trees." (Kerk, 1999b)

Objectives and organization of study
Windfall gains accrued to the owners of collective sales have been the focus of the local media. Why do the developers pay a huge premium for the collective sale sites? What are the economic values for the developers in the collective sale deals? In acquiring the freehold collective sale sites, the developers will have to pay a first-cut of development windfalls to the owners, and then a second-cut to the government in the form of development charges. The residual values, if any, based on the conventional valuation theory, will be accrued to the developers for undertaking the development and market risks. In addition, the developers will also be able to optimize the redevelopment potential of the site by having the flexibility to select the optimal time to undertake the redevelopment. The redevelopment timing option premiums may not have been captured at the feasibility stage, due to the methodological constraints of the conventional residual valuation approach.

The objectives of the paper are therefore two-fold. Firstly, this paper aims to examine the economic motivations underpinning the developers' decision in the collective sales, and also to explain the premium associated with redevelopment timing options. Secondly, it applies a discrete time binomial option-pricing model to empirically estimate the redevelopment option premiums for 30 collective sites transacted in the first ten months of 1999.

The collective sale sites were selected in our analysis for two main reasons. Firstly, most of these sites have freehold tenures. They are more flexible in conversion efficiency, ie. the frequency of redevelopment of the subject sites (Capozza and Sick, 1991), compared to leasehold sites with predominantly 99-

year leases.[4] For leasehold lands, the redevelopment potential will diminish as the lease terms shorten. The redevelopment timing option is not infinite. Secondly, collective sale sites, which are not used to the highest and best potential, are likely sites to benefit from the enhanced development density created in the development guide plan revision exercise. Redevelopment timing is thus more valuable for developers of these collective freehold sites compared to other leasehold sites.

The empirical methodology proposed in this study differs from that adopted by Quigg (1993). Collective sale sites used in this study consist of lands with redevelopment potential, and there are also interim cash flows generated from the underdeveloped structure on the lands. However, in Quigg's (1993) paper, more sample sites were collected, which consist mainly of vacant lands. The development potentials of the sample vacant sites were determined based on comparable developments in the surrounding areas, which may not be optimal at the time of analysis. In our analysis, the subject collective sale sites are assumed to be redeveloped to the maximum density permitted under the development guide plans. Prices of potential buildings to be built on Quigg's (1993) sample vacant sites are estimated from actual transaction data using hedonic pricing models. The hedonic based estimated property prices were not adjusted for timing and depreciation of the developed properties. In our study, the transactions of comparable properties surrounding the collective sale sites are employed to represent the market prices of buildings that will be constructed on the collective sale sites.

This paper is organized into six sections. Section I sets up the objectives of the study, and it gives a background overview of the collective sale activities in Singapore. Section II reviews the theoretical and empirical real option literature that examines the issues of optimal timing of development and redevelopment of lands. Section III describes the collective sale process in Singapore and the transactions taken place in 1999. Section IV explains the significance of the optimal redevelopment timing options embedded in the collective sale sites. Section V applies the binomial tree option-pricing model to price the premiums for the redevelopment timing options. The empirical estimations of the redevelopment timing premiums for a sample of 30 collective-sale sites are analyzed in Section VI. Section VII concludes the study.

LITERATURE REVIEW

The development timing option problem in real estate was first examined by Titman (1985) using a simple binomial tree model. He argued that options to wait

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[4] Government's land sale sites constitute the second major supply of the new development sites. For residential sites, all the sites will have a fixed project completion period of 4-5 years imposed as a condition of sale by tender. There was a liquidated damage of 2% of land price per month for delays beyond the stipulated project completion period prior to June 1998.

to develop undeveloped or under-developed land are valuable if the market is volatile. Williams (1991) extended Titman's model using a continuous time approach with the development value and the development cost as stochastic variables. Quigg (1993) subsequently tested Williams's (1991) model using empirical transaction data in Seattle. The average development timing option premiums were estimated at 6% of the intrinsic values of the undeveloped lands.

Sing (2001) reworked Williams (1991) and Quigg (1993) models by incorporating elasticity of scale functions to the rental and development cost variables. He then applied the proposed option model to analyze the optimal timing option for a vacant site in London. He found that based on the traditional discounted cash flow (DCF) model, which ignores the timing options, developers accept a real estate development decision at too low a cut-off rate. Sing and Patel (2001a) empirically tested the option premiums of commercial properties in the UK from 1984 to 1997 using a single factor model. The timing option premiums were estimated to range from 16.06 percent to 28.78 percent.

Yamazaki (2001), in another empirical test using land price data in central Tokyo, confirmed the proposition of the real option theory. She found that the uncertainty of built asset returns has a significant positive effect on land price. The results were also consistent with the empirical test of Sing and Patel (2001b), who found a significant negative relationship between market uncertainty and investment in real estate market. When the market was uncertain, investors would defer their real estate investment decision because the option to wait was valuable. As a result, the aggregate investment in the market declines.

Land is a durable capital. Demolition and redevelopment can be repeatedly carried out to maximize the potential value of the lands. Williams (1997) examined the effects of repeated redevelopment options on land values in a single land use framework. He showed that developer would choose to convert under-developed or undeveloped lands more frequently and less extensively, when the conversion efficiency of the lands was high. Market values of land with several redevelopment options were higher than those with only one redevelopment option. Childs, Riddiough and Trianitis (1996) extended the multiple redevelopment option models for under-developed or undeveloped lands to include two alternative land uses. The comparative statistics showed that the values of the vacant land with redevelopment potential was less sensitive to the changes in redevelopment cost than the changes in built property value.

Collective sale in Singapore
Collective sale process
Property owners, developers and property agents are the "deal-makers" in a typical collective sale. Under the Land Titles (Strata) Act (Chapter 158) 1985 and the Land

Titles (Strata) (Amendment) Act 1987[5], unanimous resolution from the owners of strata-titled property is a prerequisite for a successful collective sale. The en-bloc sale will have to be aborted as long as one of the owners is opposed to the disposal of the strata-titled development. The unanimous resolution enacted in the pre-1999 Acts holds up many potential collective sale deals. Consensus building in a collective sale is a long drawn and costly tussle, which may stretch over several weeks and months depending on the size of the development. Minority owners may also exploit the rights conferred by the Acts to demand for higher payoffs as a condition to cooperate in the transaction

The collective sale process is normally initiated by a group of enthusiastic owners, who are motivated to reap potential windfalls in their development. Developers may also, in some cases, make the first move to initiate the negotiation process with the owner representatives. However, when the market competition heated up in 1999, property agencies and brokerage firms became more pro-active in searching and identifying sites with collective sale potential. They volunteered their professional advisory services to owner representatives of potential collective sale sites hoping that they would obtain an exclusive right to broker the deal.

After the preliminary contact has been established, series of meetings between the owner representatives, the property agents and developers will be arranged. Issues raised and discussed in the negotiation process include the selling prices, the apportionment of sale proceeds, options to repurchase properties in the redeveloped property, relocation of existing owners, outlined planning permission, determination of development charges[6] and terms and conditions of sale. Property consultants would advice the owner representatives on procedural matters and recommend a reasonable reserve price for the sale.

An informal collective sale committee is formed to communicate the sale proposal to and to garner support and feedback from other owners. Next, a special general meeting is convened, where consensus from the majority of owners must be obtained so that a resolution can be passed to authorize the disposal of the strata-titled development. In the special general meeting, a collective sale sub-committee is constituted and a solicitor is appointed to provide legal advice on the sale. The sub-committee represents the owners in making decisions on the mode of sale, either by tender or auction, evaluating and negotiating offers from prospective

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[5] These two Acts govern the strata-titles subdivision and registration of the joint and common interests of a development with multiple ownerships. The acts create the strata ownerships of individual space, and at the same time, they allow joint ownership of common areas within the boundary of the strata-titled development.

[6] Development charge is a form of betterment levy imposed by the government and administered by the Urban Redevelopment Authority (URA), which allows the government to cream off 50% of the enhancement land value created by planning initiatives, such as the upgrading of the permitted zoning and density of development.

buyers, finalizing the terms and prices of the sale, and then signing the sale and purchase agreement.

The distribution and apportionment of sale proceeds is a delicate and sensitive factor that can break a collective sale deal. The net sale proceeds after deducting all expenses are apportioned based on built-up area, share value, or open market value of each unit. Disputes may arise during the negotiation stage, when some owners demand higher apportionment of the sale proceeds because they have carried out renovations to their units, or their units enjoy better view and frontage. Time taken for a collective sale commencing from negotiation till the delivery of vacant possession may vary from a few months to years. For example, in the collective sale of Changi Heights, the last payment of the sale proceeds was only released to the 58 owners nine months after signing the sale agreement.

Collective sale in 1999
The collective sale frenzy created by the Cosy Mansion and Changi Heights transactions in 1994 halted after the regional financial crisis occurred in July 1997. The stories of owners becoming millionaires overnight from the collective sales during the period of 1994 to 1996 have also disappeared. The property market benchmarked by the Urban Redevelopment Authority property price index declined by more than 36 percent between the third quarter of 1997 and the fourth quarter of 1998 (Figure 1). With the sharp decline in the property prices, some market analysts predicted that collective sales had become a thing of the past, and the magnificent performance of the collective residential sales would not likely to occur again in the next few years (Kerk, 1998).

A new wave of collective sale activities was created in the market following the passing of the Land Title (Strata) (Amendment) Act on 4 May 1999, which removed the statutory requirement to obtain unanimous consents from the owners in a collective sale. Under the amended Act, 90 percent of the owners' votes, based on their share values, would be sufficient to form a majority decision for a building that is less than ten years old, whereas 80 percent share values are required if the building is more than ten years old. These legislative amendments minimize delay in collective sale process. Coupled with the strong recovery of residential prices in 1999, developers, consultants and property analysts have shown renewed interest in collective sales (Kerk, 1999a; Tan, 1999a). A significant comeback was observed in the first ten months in 1999 with thirty collective sale sites transacted.

Figure 1: Private residential property price movement in Singapore (1986 Q2 – 1999 Q3)
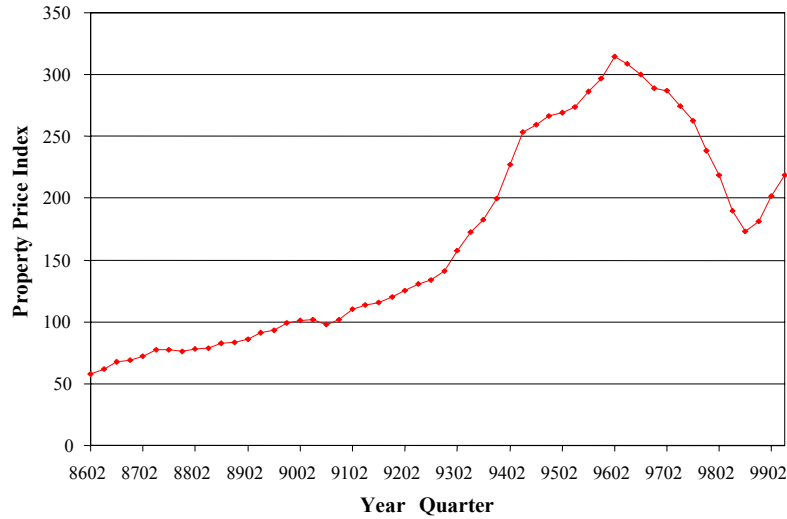
Source: The Urban Redevelopment Authority (URA) of Singapore

Optimal redevelopment timing

For a project with a gross floor area of Q and a space utilization efficient factor of γ, the gross development value (GDV) can be defined by multiplying the expected unit sale price, ($\overline{P}$), by the net saleable floor area of the development, [Q*γ], which is given below:

$$GDV = (\overline{P})*Q*\gamma \tag{1}$$

The gross development cost (GDC), on the other hand, consists of construction/ redevelopment costs (UC*Q), land cost (L) and development charge (DC), which is written as follows:

$$GDC = (UC*Q) + L + DC \tag{2}$$

where UC is the unit construction cost that includes all ancillary costs of developments. For collective sale cases, the land cost (L) is equivalent to the lump-sum acquisition price that is valued based on the redevelopment potential of the site.

In the breakeven analysis for a development project, the minimum or breakeven price (P^b) for the development is determined when the gross development value

(GDV^b) equals the gross development costs (GDC). However, when the project is put on the market for sale upon completion, the expected sale price ($\overline{P}$) may be higher than the breakeven price (P^b). The margin or the difference between two prices ($\overline{P}$ – P^b) multiplied by the net saleable area (Q*γ) will constitute the developer's profits and returns (PR) for bearing risks in the development process. The profit margin may vary from developer to developer, and from project to project. It is also dependent on the developers' risk preference, ie. the amount of risk a developer will take for a specific return on the investment.

In a deterministic framework, the developer's expected profits and returns (PR) can be defined as the difference between the expected GDV and the GDC[7], or the equivalent of the price margin multiplied by the net floor area. It can be represented mathematically as:

$$PR = GDV - GDC = (\overline{P} - P^b)* (Q*\gamma) \qquad (3)$$

The redevelopment value for a collective sale site (L) can be estimated as the residual of the GDV. If the developer's profit and return (PR) for undertaking the development risk can be *a priori* determined, the collective sale land price (L) is estimated by deducting the developer's profit and return (PR) from the net development revenue (NDV), where NDV = GDV-(UC*Q+DC). A higher land price for a particular collective sale site will therefore be translated into a smaller profit margin for the developer, and vice versa.

In a dynamic residential property market, decisions based on a deterministic price assumption are unrealistic. This deterministic approach to evaluating a project's feasibility neglects the significance of timing options in the redevelopment decision. If the future price evolution is uncertain, a positive net present value (NPV) at the point of evaluation will not be sufficient to induce developers to exercise the redevelopment options on the acquired sites. By deferring the option to redevelop by one period, the developers will be able to reap a higher NPV if the market moves upward. In contrast, if the market moves downward, by not exercising the redevelopment option earlier, the developers are protected against possible downside losses.

In a volatile market, the opportunity cost of not deferring the redevelopment decision may prove to be costly for the developer. The developer will not forgo the option to wait, unless the NPV return is sufficient to compensate him not only for taking the risks in the development process, but also for giving up the possible upside potentials that may arise in future. The additional return is the timing premium embedded in the

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[7]   The GDV and GDC in equations (1) and (2) omit the elasticity of density effects for the price and cost like those modelled by Williams (1991) and Quigg (1993).

redevelopment potential of a collective sale site. The decision of the listed City Developments Limited (CDL) to defer the redevelopment of Kim Lin Mansion, the collective sale freehold property acquired in late 1999, by leasing out all the 124 apartments instead (Kalpana, 2002), is an example of how a timing option is important to the developer.

The optimal timing premium, if properly accounted for, could be translated into a higher residual value for the collective sale sites. In other words, a developer can outbid other competitors in a collective sale site tender exercise by offering a bid price, which is higher than the residual land value estimated using a deterministic framework with the same assumptions on the developers' returns and profits. In practice, premiums could be observed in the competitive bidding exercise, and how much are the premiums associated with the optimal timing options may not be quantifiable by the bidders. They may have build-in such a premium based on their intuitive judgment of future market volatility. This paper attempts to develop a more structured way to explicitly price the option premiums embedded in 30 collective sale sites transacted in 1999 using the option pricing methodology[8].

Option pricing analogy
An option is a right, but not an obligation, to take an action in the future. A call option on a stock gives the buyer a right to buy the stock at an agreed price in a future date. The call option will only be exercised if the price of stock at the contract expiry date exceeds the agreed price. The buyer of the option will pay the underwriter an option premium at the contract date in return for the hedging benefits against the future price risks.

Black and Scholes (1973) and Merton (1973) developed a revolutionary option-pricing framework, which has been widely adopted by financial analysts and investors to price risks in the stock markets. Based on the risk neutrality principle, an option contract can be priced without the need to determine the risk aversion and expectation of the investors. For an illustration, assume that a call option is written to buy one unit of City Developments Limited (CDL)[9] share in one year time at an exercise price of say S = $8.50. The CDL share on the contract date was traded at say P= $8.15 (as on 7 April 2000).

Let us assume that the price will appreciate to P_u = $9.00 ($u = P_u/P$ =1.104) in a year's time with a probability of 0.5, and also there is an equal probability that it

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[8] There is a bumper crop of collective sales transacted in 1999 due to the change to the LTSA, which requires only 90% consensus of residents to conclude the deals. The redevelopment timing effects were effectively sampled by a cross-section of 30 collective sales used. The number of collective sales fluctuates from year to year, and by only focusing on the 1999 sample, we could eliminate the temporal effects on the redevelopment timing premiums.

[9] City Developments Limited (CDL) is the largest listed property company in Singapore with a market capitalization of S$6,047.7 million as in April 2000. It also holds a land bank of more than 0.46 million square meters (5 million square feet) of residential and commercial lands.

will go down to P_d = \$7.50 ($d = P_d/P$ =0.920) at the end of year one. The coupon yield of the Jurong Town Corporation (JTC)[10] 10-year fixed rate bonds is used as a proxy of the risk-free return, r_f = 5%. The risk-neutral probability (π) is computed using the following formula:

$$\pi = \frac{\iota - d}{u - d} = \frac{(1 + r_f)*P - P_d}{P_u - P_d} = \frac{(1 + 0.05)*8.15 - 7.50}{9.00 - 7.50} = 0.705 \qquad (4)$$

where ι is one plus risk-free rate of interest, $[1 + r_f]$, u and d are the multiplicative upward and downward movements in stock prices respectively.

At the expiration of the option, the call option would be exercised, if the price goes up to \$9.00, and the payoff for the option-holder is c_u = P – S = \$9.00 - \$8.50 = \$0.50. On the other hand, if the price drops to \$7.50, the option will not be exercised [c_d = \$0]. The premium for the call option, (c), on the CDL share if tradable in the market is estimated as follows,

$$c = \frac{\pi * c_u + (1 - \pi) * c_d}{(1 + r_f)} = \frac{(\$0.50 * 0.705)}{1.05} = \$0.34 \qquad (5)$$

where c_u is the option payoff when the asset price moves up by u, and the option payoff c_d is worthless when price declines below the strike price, S. Using the call option analogy, the acquisition of a collective sale site is theoretically equivalent to purchasing a redevelopment option on the existing site. The option will only be exercised if and only if the gross development value of the redeveloped project exceeds the land acquisition cost (ie. cost of collective sale property) and development costs. The development costs comprise the construction costs, financing costs, holding costs, developer's return and profits and also the interim rental income that will be forgone if the redevelopment option is exercised.

In the traditional breakeven analysis, the developer will activate a project and launch the pre-completion sale[11] as long as the market price is above the breakeven point. The margin between the selling and breakeven prices will determine the developer's returns and profits for undertaking the redevelopment project. There is no timing risk assumed by the developers as they pre-sell their projects and lock the price to current market price.

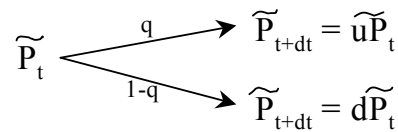[10] Jurong Town Corporation (JTC) is a government's statutory board responsible for the physical planning, development and management of the public industrial facilities in Singapore.
[11] There is a time lag between activation of the construction and pre-sale in a typical development project. However, in this context, we omit the time-lag factor and assume that the construction starts and the pre-sale occur concurrently.

However, in a dynamic residential market, the optimal timing to start a project is strategically important for a developer. In an upward market, if the developers keep the option to redevelop, ie. defer the redevelopment plan, they may reap the upside benefits if the market price moves up substantially after the project has been launched. On the other hand, if the market declines, the downside risks are protected because the developers will not be forced to sell the projects at loss.[12] A discrete time binomial-tree redevelopment option model is proposed to determine the value of the options for waiting to redevelop the collective sale sites.

Binomial tree redevelopment option model
The discrete-time binomial tree option-pricing framework proposed by Cox, Ross and Rubinstein (1979) is applied and adopted to price the redevelopment option premiums embedded in the collective sale sites. In the proposed model, we assume that the expected selling price of the potential property to be redeveloped on the collective sale site, denoted as $\widetilde{P}$, is the sole stochastic state variable, which follows a discrete binomial tree process. Over a discrete interval of dt, $\widetilde{P}$ will move either upward by u with a probability of q, or downward by d with a corresponding probability of $(1-q)$:

$$\widetilde{P}_t \quad \xrightarrow{\quad q \quad} \quad \widetilde{P}_{t+dt} = \widetilde{uP}_t$$
$$\xrightarrow{\quad 1-q \quad} \quad \widetilde{P}_{t+dt} = \widetilde{dP}_t$$

The binomial price generating process would be extended in a triangular tree pattern over a period of T. By imposing a non-arbitrage condition $[u > \iota > d]$ on the model structure, where ι is one plus the risk-free interest rate (r_f), we can then estimate the expected property price at the end of period $[t+dt]$ as follows:

$$\iota P_t \quad = \quad uP_t\pi \quad + \quad dP_t(1-\pi) \tag{6}$$

By re-arranging the above non-arbitrage price equation, the risk-neutral probability (π) can be defined as follows:

$$\pi \quad = \quad \frac{\iota - d}{u - d} \tag{7}$$

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[12] By deferring the development decision, the developer will incur holding costs on the land unless the land is acquired fully by equity. The value of the option to wait to develop will be subject to the holding costs, which however are not considered in this paper.

The underdeveloped building sitting on the collective sale land may generate interim rental income (R_t) to the option holder during the periods prior to exercising the redevelopment option. The interim income from the existing underdeveloped site will have to be forfeited once the redevelopment option is exercised. Therefore, when assessing the feasibility of the redevelopment option, developers will have to take into consideration the loss of the interim rental income. Two deterministic growth rates (α_1 and α_2) are incorporated to reflect the inflationary effects of the GDC components and also the interim rental income. The value-matching condition (Sn) that triggers the redevelopment option at period n can be defined as follow:

$$S_n = (UC_{t=0}Q + L_{t=0} + PR_{t=0}) * (1+\alpha_1)^{ndt} + DC + R_{t=0}(1+\alpha_2)^{ndt} \qquad (8)$$

The payoff function (C) for the redevelopment option at maturity, which is assumed to be T = 5 years, is defined as the difference between the GDV_T and the S_T. The option payoff function is written as follows,

$$C(T, T\text{-}dt, i_T=0, j) = GDV_T - S_T = \text{Max}\ [(\widetilde{P}_0\ Q\gamma\ u^j d^{(n-j)}) - S_T,\ 0] \qquad (9)$$

where i indicates the order of computation from the terminal period T, ie. $i_T = 0$, to the origination, ie. $i_0 = 5$, j indicates the vertical position of the binomial tree node at each period of time, such that $j = 0, 1, 2, \ldots n$, and $n = T/dt$, where dt is one year in our analysis.

The redevelopment option is an American option, which is exercisable at any time before maturity in time T. The premium for the redevelopment option can then be worked out recursively down the binomial tree from $i_T = 0$ to the initial node at $i_0 = 5, j=0$ using the following generalized option formula,

$$C(T, t=0, i=5, j=0) =$$
$$\sum_{i=1}^{n}\sum_{t,j=0}^{n-i} Max\left\{\left(\widetilde{P}_0 Q\gamma\ u^j d^{n-i-j} - S_t\right), \left[\pi\ C(n,i-1,j+1) + (1+\pi)C(n,i-1,j)\right]* t^{-dt}\right\} \qquad (10)$$

The first part of the maximization condition in equation (10) suggests that the GDV of the collective sale site should at least be greater than or equal to the GDC before it is optimal for redevelopment to proceed. The second part of the condition represents the weighted value of the option payoffs in one period ahead. The maximum operator will choose the higher of two values between the first and second conditions as the option premium for the period in consideration.

There are limitations in the proposed redevelopment option model for the collective sale sites. Firstly, the time-to-built option feature as proposed by Majd and Pindyck (1987) is not incorporated in the model. In other words, we assume that the new building on the collective sale site will be built instantly upon exercising the redevelopment option. With this assumption, the cost of investment and revenue will be realized in the cash flow analysis at the point the redevelopment option is

exercised[13]. Secondly, the proposed model allows only one redevelopment option for each collective sale site[14]. It assumes that the option will be exercised within a maximum period of five years. These assumptions on the number of redevelopment options and also the period within which the redevelopment option has to be exercised, impose a floor on the redevelopment option premium for the collective sale sites. For simplification of the model specifications, the proposed model omits variables like taxes, transaction costs, financing costs for the acquisition and re-construction of the collective sale sites, and demolition costs.

VALUE OF WAITING TO REDEVELOP

Data

The optimal redevelopment timing option premiums are estimated empirically using a sample of 30 collective sale sites transacted in the first ten months of 1999. The expected prices of property that will be built on the collective sale sites are assumed to move randomly following a binomial tree path. For the empirical analysis, we fix the parameters for selected input variables, which include u, d, r_f, α_1, α_2, T and γ. The prices in one period ahead will move either upward by 10% (u = 1.1) or downward by 10% (d = 0.90), the risk-less interest rate (r_f) is taken at 4%, the inflationary growth rate for the GDC components (α_1) is assumed at 3%, and the interim rental growth is constant, ie. $\alpha_2 = 0\%$. The time to maturity (T) of the redevelopment options is limited to a maximum of five years, which means that the redevelopment option will be exercised anytime within the next 5 years. The gross development value is estimated based on a space efficiency factor (γ) of 85%. We assume that the developers will expect a profit and return (PR) of 15% of the GDV for undertaking the redevelopment risk and this PR figure does not take into account the optimal timing premium[15]. This PR term constitutes a part of the strike price of the proposed redevelopment option model as in Equation 8[16]. It has a negative effect on the time to wait to develop. It implies that when the rate of the profit and return is high, there is less incentive for the developer to defer the redevelopment decision.

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[13] The time-value of money factors are not explicitly incorporated for the GDV and GDC variables, because all the cash flows incurred or received at time t=0, which is the point when the redevelopment option is exercised. The time-value factor, however, has been incorporated in the option premium computation, where the option premiums at different point t are discounted by the risk-free interest rate iteratively back to time zero.

[14] Technically, the assumption can be relaxed to allow multiple redevelopment options that are exercisable over the entire life of the collective sale sites, which contain mainly 999-year and freehold leasehold tenure.

[15] By making the profit and return (PR) variable explicit and constant in the model, we could distinguish and separate the option premium from the returns which are associated with undertaking the redevelopment risks.

[16] The assumption of a constant PR rate of 15% across the 30 sample collective sale sites may be somehow naïve and constrained. However, the constant PR rate is only intended to control for development risks of the project in this model, so that the timing premium could be distinguished in the computation.

Table 1: Input parameters for estimation of redevelopment option premiums

No.	Name of Collective Sale Site	Land Area (sqm)	Unit Sale Price ($sqm)	Gross Development Value (S$)	Profits & Returns (S$)	Construction Cost (S$)	Development Charge ($)	Annual Rental ($)
1	Goldenhill Condo, Lorong Chuan	24,349	7,532	327,369,000	49,105,350	74,277,000	35,000,000	3,420,000
2	Kim Lim Mansion, Along Grange Road	11,664	18,292	380,854,026	57,128,104	59,302,530	11,400,000	3,720,000
3	First Mansion	10,925	10,222	265,789,594	39,868,439	44,435,412	16,000,000	2,721,600
4	Meyer Tower	7,296	10,222	177,508,849	26,626,327	29,676,402	11,500,000	1,573,200
5	Arang Court in Serangoon Ave 3	11,186	7,532	150,394,818	22,559,223	34,123,194	20,000,000	1,612,800
6	Newton Point, Newton Road	4,273	11,836	120,359,932	18,053,990	19,309,080	7,750,000	1,209,600
7	Devonshire Court	5,407	15,064	193,855,760	29,078,364	24,435,600	402,000	2,455,200
8	Ixora Court and surrounding land parcels	4,016	13,988	133,701,022	20,055,153	18,149,460	6,700,000	1,075,200
9	Country Park in Bedok	14,797	8,070	142,102,958	21,315,444	30,092,391	16,500,000	1,214,400
10	Cuscaden Tower, Cuscaden Walk	3,570	18,292	155,414,952	23,312,243	24,199,560	9,000,000	2,037,600
11	Grange Mansion	5,093	16,678	151,612,367	22,741,855	25,892,055	500,000	1,774,080
12	Devt Site, Devonshire Rd	4,038	16,140	155,123,640	23,268,546	18,249,840	7,780,000	1,284,000
13	Dragon Pearl n River Valley View	3,751	12,374	110,459,846	16,568,977	16,950,360	200,000	1,440,000
14	Duchess Park and Adjacent Bungalow	7,268	12,643	109,344,548	16,401,682	16,422,210	7,000,000	1,214,400
15	Mediterranean Townhouses in Bukit Timah	4,149	10,222	50,465,520	7,569,828	8,436,960	3,000,000	1,056,000
16	Naga Court, off Bukit Timah Road	4,611	15,064	123,995,382	18,599,307	15,629,670	500,000	950,400
17	Buckley Mansion, Buckley Rd	4,098	13,988	68,214,965	10,232,245	9,259,950	3,600,000	1,214,400
18	Duchess Road Semi-d and Bungalows	4,347	12,643	65,397,551	9,809,633	9,821,910	6,500,000	1,080,000
19	Mandalay Court, Moulmein	4,515	9,146	98,281,386	14,742,208	18,363,996	200,000	1,548,000
20	5 Balmoral Park	3,518	16,140	77,230,320	11,584,548	9,085,920	6,000,000	633,600
21	Chen Yuan Building, River Valley Close	1,889	12,374	55,640,473	8,346,071	8,538,180	132,300	712,800
22	16-18A Buckley Rd	4,216	13,988	70,179,655	10,526,948	9,526,650	3,700,000	450,000
23	Balmoral Haven, Balmoral Crescent	3,298	16,140	72,383,280	10,857,492	8,515,680	6,320,000	534,230
24	18-28 Ipoh Lane	2,567	8,608	52,599,904	7,889,986	10,442,628	4,000,000	324,000
25	Balestier Mansion	3,273	7,801	60,775,561	9,116,334	13,313,916	141,000	1,116,000
26	Devt Site in Shanghai Road	2,377	12,374	52,507,292	7,876,094	8,057,385	8,000,000	420,000
27	Norfolk Garden	2,910	10,222	70,796,432	10,619,465	11,835,936	119,000	1,108,800
28	Sunshine Apts, along Upper Serangoon	1,943	7,532	34,837,726	5,225,659	7,904,358	1,700,000	330,000
29	Devt Site, off Balmoral Crescent	1,496	12,912	26,268,672	3,940,301	3,863,040	2,700,000	300,000
30	Surrey Ville, Surrey Rd	704	12,374	20,719,090	3,107,864	3,179,400	157,800	268,800

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Based on comparable market evidence, we also project the expected sale prices ($\widetilde{P}$), unit construction costs (UC), land cost (L), development charge (DC), developers' profits and return (PR) and the interim rentals receivable before redevelopment (R) for all the 30 sample collective sale sites. The expected selling price ($\widetilde{P}$) for the property on the redeveloped site at period t = 0 is determined based on the comparable sales evidence of new condominiums in the vicinity collected from the on-line database of the Urban Redevelopment Authority (URA). The unit construction cost estimates (UC) were obtained from a local quantity surveyor firm. The unit construction cost is pegged to the type of the proposed development, categorized into three classes: luxury ($\geq$S$16,140 psm), above average ($\geq$S$10,760 psm and <S$16,140 psm) and average (<S$10,760 psm). The unit construction costs for the luxury, above average and average class of condominium development are estimated at $2,421 psm, $1,614 psm and $1,453 psm respectively. Development charge (DC) is computed based on a fixed rate table compiled by the tax authority of Singapore's government - the Inland Revenue Authority[17]. The interim rentals (R) for the collective sale properties prior to the redevelopment are estimated based on the prevailing market rental, after adjusting for factors like the unit size of the property, the property's age, location and facilities in the development. The details of input parameters for the collective sale sites are given in Table 1.

Analysis of results
The redevelopment option of the collective sale site will be exercised if the gross development value exceeds the exercise price, which consists of the land costs, development charge, construction costs, developer's profits and returns, and loss of interim rental income. The payoffs or gains from exercising the redevelopment options are computed in a backward iterative process from year five to year 0. The average premium for the redevelopment timing option was estimated at S$12.24 million or 9.65% of the gross development value. The results imply that the collective sale sites will not be redeveloped unless the returns from the sale of the redeveloped site are large enough to pay off the developers' risks for undertaking the development process (15%) and also to compensate the developer for forgoing the option to wait for the upside of the market (9.65%), which is estimated additively at 24.65% of gross development value.

The option premiums for the 30 sample collective sale sites were estimated and ranked in a declining order in Table 2. In percentage terms, the option premium estimates vary within a range between 5.79% and 15.61%. The results showed that the Mediterranean townhouse collective sale site at Bukit Timah Road has the highest redevelopment option premium of 15.61%, whereas the Norfolk Garden site has the lowest premium of 5.79%.

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[17] The development charge payable for the proposed development is reported together with the source of the collective sale transactions.

The results seem to show discernible relationships between the land size and collective sale price and the optimal redevelopment-timing premium. The results show that Goldenhill Condominium located at Lorong Chuan with the largest land area of 24,349 square meter (sqm) has the highest option premium of S$43.50 million, whereas Kim Lim Mansion site with the highest collective sale price of S$251 million has the second highest option premium of $43.39 million. On the other hand, the smallest site with the lowest collective sale price of S$11.50 million, Surrey Ville, has the lowest option premium of $3.08 million. To verify the above relationships, we run a regression of the redevelopment timing option premium (C) on the land size (Area) and collective sale price (L) for the sample of 30 sites. The regression result is shown below with the t-statistics in the brackets:

$$ C = -2,427,057 + 699.19 * Area + 0.153 * L + \varepsilon \quad (11) $$
$$ (5.151) \quad\quad (12.643) $$

where ε is the regression error term. The regression shows that the coefficients for the two independent variables, Area and L, are positive and significant at a 5% level. It implies that the redevelopment timing option premiums for the collective sites are linearly and positively related to the size and also the price of the collective sale sites.

Sensitivity analysis
Sensitivity analyses are conducted to examine the effects of changes in risk-free interest rate (r_f), the multiplicative upward (u) and downward (d) price movements on the option premiums. The risk-free rate varies within a range between 1% and 8%. The multiplicative upward and downward movements of prices are varied over the following ranges, *u:* 1.10 to 1.40 and *d:* 0.65 to 1.00. The spread between *u* and *d* represents the market volatility. Results of the sensitivity analysis are summarized in Table 3.

We found that the premium for the redevelopment timing option is highly sensitive to the magnitude of the upward and/or downward movements of property prices in the future. The results show that the redevelopment option premium increases when the multiplicative price spread increases resulted either by an increase in *u* or a decrease in *d*. The price spread, which measures the price volatility shows a positive relationship with the premiums of redevelopment timing option. When *u* increases from 1.10 to 1.40 and d is fixed at 0.90, the option premiums (C) increase from S$12.24 million (9.65%) to S$23.71 million (18.97%). When we reduce the price spread from 0.45 to 0.15 by varying *d* from 0.65 to 0.95 at a fixed *u* of 1.10, the option premium declines from S$20.30 million (16.19%) to $10.37 million (8.21%). The relationship between the option premium and the risk-free interest rate is also positive. When the risk-free interest rate increases from 2.00% to 8.00%, the corresponding option premiums increase from S$7.27 million (5.33%) to $25.74 million (21.20%). The results of the sensitivity analysis are consistent with the propositions in the conventional option theory.

Table 2: Option premiums for redevelopment of collective sale sites

No.	Name of Collective Sale Site	Land Area (sqm)	Plot Ratio	Built-up Area (sqm)	Collective Sale Price ($)	Option Premium (S$)	(% of GDV)
1	Goldenhill Condo, Lorong Chuan	24,349	2.1	51,133.8	175,800,000	$43,497,470	13.29%
2	Kim Lim Mansion, Along Grange Road	11,664	2.1	24,495.1	251,000,000	$43,488,213	11.42%
3	First Mansion	10,925	2.8	30,590.3	176,000,000	$36,900,439	13.88%
4	Meyer Tower	7,296	2.8	20,429.9	117,000,000	$24,697,176	13.91%
5	Arang Court in Serangoon Ave 3	11,186	2.1	23,491.1	72,810,000	$18,274,722	12.15%
6	Newton Point, Newton Road	4,273	2.8	11,963.5	78,000,000	$15,734,506	13.07%
7	Devonshire Court	5,407	2.8	15,139.8	123,000,000	$14,561,485	7.51%
8	Ixora Court and surrounding land parcels	4,016	2.8	11,245.0	84,000,000	$13,293,276	9.94%
9	Country Park in Bedok	14,797	1.4	20,716.2	65,500,000	$13,030,982	9.17%
10	Cuscaden Tower, Cuscaden Walk	3,570	2.8	9,995.7	86,000,000	$12,783,609	8.23%
11	Grange Mansion	5,093	2.1	10,694.8	92,000,000	$12,394,198	8.17%
12	Devt Site, Devonshire Rd	4,038	2.8	11,307.2	92,000,000	$12,202,142	7.87%
13	Dragon Pearl n River Valley View	3,751	2.8	10,502.1	73,500,000	$11,284,431	10.22%
14	Duchess Park and Adjacent Bungalow	7,268	1.4	10,174.9	61,300,000	$9,172,567	8.39%
15	Mediterranean Townhouses in Bukit Timah	4,149	1.4	5,808.2	34,600,000	$7,877,475	15.61%
16	Naga Court, off Bukit Timah Road	4,611	2.1	9,683.8	72,120,000	$7,868,669	6.35%
17	Buckley Mansion, Buckley Rd	4,098	1.4	5,737.3	43,000,000	$7,332,730	10.75%
18	Duchess Road Semi-d and Bungalows	4,347	1.4	6,085.4	37,000,000	$7,078,330	10.82%
19	Mandalay Court, Moulmein	4,515	2.8	12,642.2	52,000,000	$6,588,055	6.70%
20	5 Balmoral Park	3,518	1.6	5,629.4	42,000,000	$5,966,598	7.73%
21	Chen Yuan Building, River Valley Close	1,889	2.8	5,290.1	37,500,000	$5,931,743	10.66%
22	16-18A Buckley Rd	4,216	1.4	5,902.5	40,600,000	$5,586,099	7.96%
23	Balmoral Haven, Balmoral Crescent	3,298	1.6	5,276.1	38,300,000	$5,575,411	7.70%
24	18-28 Ipoh Lane	2,567	2.8	7,188.9	28,600,000	$5,377,193	10.22%
25	Balestier Mansion	3,273	2.8	9,165.6	33,400,000	$4,893,370	8.05%
26	Devt Site in Shanghai Road	2,377	2.1	4,992.2	23,800,000	$4,694,607	8.94%
27	Norfolk Garden	2,910	2.8	8,148.1	35,700,000	$4,097,772	5.79%
28	Sunshine Apts, along Upper Serangoon	1,943	2.8	5,441.5	18,800,000	$3,509,684	10.07%
29	Devt Site, off Balmoral Crescent	1,496	1.6	2,393.5	13,100,000	$2,177,899	8.29%
30	Surrey Ville, Surrey Rd	704	2.8	1,969.9	11,500,000	$1,375,731	6.64%

Table 3: Sensitivity analysis of redevelopment option premiums

Upward movement	1.10	1.15	1.20	1.25	1.30	1.35	1.40
Downward movement							*d = 0.90*
Price Spread (volatility)	0.20	0.25	0.30	0.35	0.40	0.45	0.50
Average Option Premium (S$)	$12,241,553	$15,022,690	$17,095,255	$18,569,310	$20,503,143	$22,362,214	$23,709,878
Average Option Premium (% of GDV)	9.65%	11.95%	13.52%	14.85%	16.49%	17.93%	18.97%
Upward movement							*u = 1.10*
Downward movement	0.65	0.70	0.75	0.80	0.85	0.90	0.95
Price Spread (volatility)	0.45	0.40	0.35	0.30	0.25	0.20	0.15
Average Option Premium (S$)	$20,299,281	$18,185,426	$16,335,352	$14,962,083	$13,939,907	$12,241,553	$10,372,363
Average Option Premium (% of GDV)	16.19%	14.38%	12.99%	12.06%	11.10%	9.65%	8.21%
Risk-free interest rate (r_f)	2.00%	3.00%	4.00%	5.00%	6.00%	7.00%	8.00%
Average Option Premium (S$)	$7,269,928	$9,413,234	$12,241,553	$15,323,600	$18,627,202	$22,113,932	$25,739,743
Average Option Premium (% of GDV)	5.33%	7.18%	9.65%	12.32%	15.16%	18.13%	21.20%

CONCLUSIONS

The conventional investment analysis techniques developed on the rigid "*invest now or never*" rule underestimate the strategic flexibility associated with the timing of redevelopment. The option to wait to develop/redevelop has value when the future price movement is uncertain. An option pricing analysis is able to extend the investment horizon to evaluate investment opportunity now and the periods ahead by evaluating explicitly the future price evolution process.

This paper applies the discrete-time binomial tree option-pricing framework proposed by Cox, Ross and Rubinstein (1979) to evaluate redevelopment options embedded in a sample of 30 collective sale sites in Singapore. In the redevelopment of collective sale sites, developers have an option to wait to redevelop the site if the property prices are expected to rise in the periods ahead. Option to redevelop will only be exercised when the return on the investment is large enough to compensate for giving up the future upside potential of the site. Based on 30 collective sale transactions recorded in 1999[18], our empirical results showed that the average redevelopment option premium was estimated at S$12.24

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[18] The small size of the collective sale samples and also the difficulty in empirically estimating the actual parameters for the state variables such as the price volatility are limitations in the study.

million or 9.65% of the gross development value. By adding the redevelopment-timing premium to the developer's profit and returns of 15%[19], the developer would expect a premium of 24.65% of gross development value if the option to redevelop were to be exercised now. Otherwise, it will be more valuable to keep the redevelopment option "alive." The regression results further showed that the option premiums are positively and significantly related to the size and the price of the collective sale sites.

Sensitivity analyses have been carried out and the results are consistent with the predictions of conventional option theory. The results showed that the premiums of the redevelopment option increase when the price volatility, which is measured by the spread between u and d, increases. When u increases from 1.10 to 1.40 and d is fixed at 0.90, the option premiums (C) increase from S$12.24 million (9.65%) to S$23.71 million (18.97%). Changes in risk-free interest rate have positive effects on the option premiums. When risk-free interest rate increases from 2.00% to 8.00%, the corresponding option premiums increase by more than 2.5 times from S$7.27 million (5.33%) to $25.74 million (21.20%).

With the timing premium in mind, developers will be better able to evaluate explicitly the redevelopment options when bidding for a collective sale site. By translating the option premium into the bid price for a collective sale site, the developer will stand a better chance of out-bidding the competitors, who neglect the redevelopment timing premiums embedded in the site. By recognizing the value of the timing option in a highly volatile market, developers are also better able to exploit the strategic advantages of deferring the redevelopment of the collective sale sites, especially in a down-market.

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[19] The assumption does not significantly violate the estimation of the redevelopment option-timing premium. We could, however, easily include different rates for different projects, if they are observable. In practice, developers will not normally reveal their expected rate of returns and profits in undertaking a particular project.

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