Diversification Effect of Introducing Australian and US Housing Property into Finnish Housing Property Portfolio

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Abstract

In this paper we show that a Finnish investor can benefit substantially by introducing Australian and US housing property into his housing property portfolio. We use historical total return data on a few large towns and cities in Finland and Australia and on a few states in the United States. The basis for the analysis lies in modern portfolio theory. The empirical results show that correlation coefficients between American, Australian and Finnish areas are small measured in Finnish currency. Because of this the mean-variance opportunity set moves substantially by introducing Australian and US housing property into a Finnish housing property portfolio. However, the high transaction and information costs are relevant. Furthermore, the analysis shows that exchange rate fluctuations are a major factor influencing foreign housing property returns and standard deviations expressed in the Finnish currency.

1. Background, objective and data

Previous empirical evidence shows that regional diversification in housing property investments is not possible in Finland: because the correlation coefficients between housing investment returns in the major centers in Finland are very high (Oikarinen 1998, 65-70). This means that in order to diversify the risk in a housing property portfolio, a Finnish investor has to add foreign housing properties into the portfolio.

The returns on housing properties are largely determined by the country's economical conditions. The Finnish economy is closely tied to European countries especially to Germany, Sweden and Russia. For the Australian economy on the other hand the economical performance of Asia is important. Therefore one would expect that the correlations between the relevant economic factors in Finland and in Australia are quite low. Thus, also the correlation between housing property returns in these two countries would be low eventhough the economics in both of these countries are closely related to the economy of United States. Therefore one can argue that adding Australian housing property into a Finnish housing property portfolio would have a significant diversification effect and move the mean-variance opportunity set substantially. Furthermore one could think that the correlation between housing property returns in United States and Finland or Australia is larger than the correlation between Finnish and Australian housing property returns. This would suggest that Australian property is a better investment for a Finnish housing property investor in terms of diversification. One purpose of this paper is to find out whether this really is the fact.

The question about diversification by investing to foreign housing property has been raised lately because of the recent forming of the economic and monetary union (EMU). Finland has been chosen to represent the EMU area in this research because we had data available from main centres in the country. Areas in Australia and in the United States in turn have been chosen for a particular reason. As Finland is an EMU member it will not have anymore any exchange rate effects with any of the other EMU member countries. This in turn decreases the attractiveness of housing property in these countries as a diversification method from Finnish investors points of view. This is caused by the fact that exchange rate fluctuations are likely to lower the correlation between real estate returns in any two EMU countries when measured in the same currency (see Newell – Webb 1996). As exchange rate variability still remains between the Finnish and Australian/US currencies, the relative attractiveness of Australian and US housing property to Finnish investor has increased during the last couple of years.

1.1 Objective

The objective of this paper is to test if a Finnish investor can benefit by introducing Australian and US housing property into his housing property portfolio. The secondary objective is to examine how much of the diversification comes from property specific factors and how important is the exchange rate effect. The basis for the analysis lies in the modern portfolio theory. The analysis is from a Finnish investor's point of view. Real returns are used most of the time in the analysis.

The methods used are basic ones in finance. There are also some problems with the data obtained. However the purpose of the paper is mainly to be a preliminary research on the topic. The final goal is to raise questions and thoughts about possible consequences of the EMU in a housing property investor's viewpoint. Subsequently, as we get more reliable data and information from other European (and possibly for example Asian) areas we are able to make deeper and more reliable analysis.

2 Modern portfolio theory and housing property investments

Modern portfolio theory (MPT) is based on the fact that by diversifying the portfolio into multiple assets one can achieve the same expected return with lower risk or a higher expected return at the same risk level. This diversifying effect exists because the returns on different assets do not correlate perfectly with each other. This is due to the unsystematic factors that influence the returns. In modern portfolio theory risk is measured by the variance or standard deviation of the returns. The formula for the variance of portfolio return is below (See Markowitz 1959).

(1)
$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{i=j}^{N} w_i w_j \mathrm{COV}_{ij}$$

$COV_{ij} = COR_{ij} * \sigma_i \sigma_j$
COR_{ij} = correlation coefficient between assets i and j
$\sigma_{i/j}$ = standard deviation of asset i/j
$w_{i/j}$ = proportion of investors wealth invested in asset i/j
$_p^2$ = portfolio variance

As can be noted the lower the correlation coefficients the smaller the risk. That is the lower the correlations between the assets, the better is the diversification because the expected return on the portfolio is simply the weighted average of the asset returns in the portfolio.

The use of variance or standard deviation as a measure of risk for property investment has some problems. Firstly, variance doesn't take the liquidity risk into account. There are significant differences in liquidity between different areas and different properties which are likely to affect the risk premium and thereby the required rate of return. Secondly, in international comparison the use of variance as the only risk measure is problematic, because it doesn't take country specific risks (e.g. political risk) into account. Nevertheless, variance is still the risk measure that is usually used in real estate analysis and in this paper, too.

Modern portfolio theory assumes that investors focus only on expected return and variance. It is possible to construct portfolios that maximize expected return given the variance or alternatively minimize variance given the expected return (Markowitz 1959). These kind of portfolios produce the mean-variance opportunity set of portfolios given available assets.

The analogue of diversifying portfolio risk applies also to housing property investments although there are some restrictions. First, MPT assumes that there are not any transaction or information costs but in real life these costs are substantial in property investments. Second, the theory assumes that all the assets are perfectly dividable. However, properties are very large (expensive) and undividable units. Because of this a large amount of capital is needed in order to diversify a property investment portfolio (see for example MacLeary – Nanthakumaran 1988, 120). However these problems eliminate the diversifying possibilities however, only partially and there is consensus among studies examining the issue that diversification in real estate investments can enhance risk-adjusted portfolio yield (see Norman et. al 1995).

3 Data

There are some problems concerning the validity of property price information. Firstly, it is often hard to find relevant price information because there is no public or centered trading place for properties. Secondly, even if one can get the information it may be biased because of thin trading and the heterogeneity of properties. In this paper the former problem has been solved by obtaining price and rental data from centered information gathering organisations. The latter problem has been solved by using four-quarter moving averages of the prices in computing the return time series. Transaction costs and taxes are ignored in the analysis. The return time series are constructed from the first quarter of 1988 to the last quarter of 1999 such that there are 48 quarters in the sample.

As common in property return analysis there are still some problems left concerning the data used. For example the housing property data we have received from the three countries differ from each other. Despite the differences we expect that the data used measures the housing property performance adequately in each area and we may quite safely perform the analysis.

The price and rental data for four Australian cities, Sydney, Melbourne, Perth and Adelaide has been received from The Real Estate Institute of Australia Ltd (REIA). The house price and rental price data are based on the sales and rental data of privately owned houses that are regularly provided to the REIA by many real estate agency firms and government and private organisations within the states and territories. Flats are not included in the sample since only a small percentage of Australians live in apartment buildings – privately owned housing data is a good measure of Australian housing investment performance.

The nominal quarterly housing property return time series for Australian cities has been constructed using the following formula.

(2) $r_t = (P_t + NR_t) / P_{t-1} - 1$

r_t = nominal return in quarter t
P_t = moving quarterly median house price in quarter t
$NR_t = wr_t * 13 * 0.8$ = net rent in quarter t
wr_t = median weekly rent in quarter t

The assumption is that the net rent (NR) is 80% of the gross rent, that is the running costs are 20% of the gross rent. The assumption is based on the estimation of Australian real estate professionals. Another assumption is that there are 13 weeks per each quarter.

The house price, rental and running cost data used to compute the housing property returns in four Finnish centres, Helsinki, Turku, Tampere and Oulu has been collected by the Finnish Statistics. Price data is based on the information gathered from major real estate agencies and is provided quarterly. However, the rental and cost figures are only reported for the second period of each year. The rental information is based on survey among the landlords and running cost data is based on the financial statements statistics of housing corporations in Finland. We have estimated the rental and running cost figures for the first, third and fourth quarter in each year as follows:

(3) average monthly rent in III quarter in year t, M(q3) = M(q2) + [M(q6) – MR(q2)]/4

(4) average monthly rent in IV quarter in year t, M(q4) = M(q3) + [M(q6) – M(q2)]/4

(5) average monthly rent in I quarter in year t+1, M(q5) = M(q4) +[M(q6) – M(q2)]/4

,where

M(q2) = the average monthly rent in the II quarter in year t

M(q6) = the average monthly rent in the II quarter in year t+1

The same formulas are used for costs but in this case the cost figures are used instead of the average rent figures.

Even though the real rents and costs may differ slightly from our estimates the difference is so small that it does not cause any significant problems, knowing that the total return on housing property was dominated by the price (not rental or cost) changes in the sample period, not by rental or cost changes.

The formula for computing nominal housing property returns in Finnish cities differs from the one used to compute the returns in the Australian data. This is because of data available in Finland is different from the data available in Australia. The return time series for Finnish cities is computed using the following formula.

(6) $r_t = [P_t + (MR_t - C_t)*3] / P_{t-1} - 1$

Pt = moving quarterly arithmetic average house price / m^2 in quarter t
MR_t = average monthly rent / m^2 in quarter t
C_t = average monthly running cost / m^2 in quarter t

Unfortunately we do not have data from cities in the United States so only state level data have to be used. We use the indices computed by the National Council of Real Estate Investment Fiduciaries (NCREIF) to measure the housing property performance in four states, California, Texas, Florida and Virginia in the United States. The method used to compute the total return differs from the Finnish and Australian data: the change in average house prices is determined partly by appraisal. This means that the standard deviation of the computed returns is likely to be smaller than the standard deviation of the housing property return really is[1]. However as moving averages are used in Finnish and Australian house prices we are able to compare the standard deviations and returns on a relatively fair basis.

The nominal return series are deflated by the consumer price index to obtain real returns. Furthermore, quarterly exchange rates are used to convert the return series into the Finnish currency. Geometrical averages of daily rates during the quarters are used.

[1] About appraisal-smoothing effect see for example Newell – Webb (1996)

4. Analysis

The analysis is made from a Finnish investor's point of view. Only real returns are used in the analysis because an investor is interested only in real returns on the capital.

Table 1 shows that there have been substantial differences between the housing property performances in the different areas during the sample period.

Table 1. Quarterly total housing property returns (arithmetic average) in real terms with standard deviations in 1988-1999

Area	**Real total return**	**Standard deviation**
Helsinki	0.0143	0.0443
Turku	0.0152	0.0368
Tampere	0.0184	0.0382
Oulu	0.0196	0.0300
Area	**Real total return**	**Standard deviation**
Sydney	0.0226	0.0352
Melbourne	0.0226	0.0226
Adelaide	0.0164	0.0125
Perth	0.0250	0.0316
Area	**Real total return**	**Standard deviation**
California	0.0194	0.0296
Texas	0.0091	0.0181
Florida	0.0114	0.0114
Virginia	0.0131	0.0198

In three out of four Australian cities the average return is higher than in any of the areas in Finland. Furthermore, the standard deviations are low in Australian cities compared to the standard deviations of Finnish cities. In other words, as an investment Australian housing property has been more profitable and less risky than Finnish housing property during the sample period. The plausible reason for this is that Finland experienced a deep slump during the sample period, in some years the Finnish GNP even decreased substantially. Also in United States the standard deviations are much smaller than in Finland but in three out of four (the exemption is California) states the average real total return has been lower than in Finnish centres. The perceived average real quarterly returns varied in the sample period from 0.9% (Texas) to 3.52% (Sydney) and the standard deviations varied from 4.43%

(Helsinki) to 1.14% (Florida).

The correlations of the returns between the Finnish cities are very high in the period such that the lowest correlation in Finland is 0.90. Also in Australia the correlations between Sydney, Melbourne and Perth (from 0.60 to 0.73) and in United States the correlations between California, Florida and Virginia (from 0.53 to 0.81) are relatively high although much lower than in Finland. But as table 2 displays, Adelaide in Australia and Texas in the US differ significantly from the other areas — the correlation between the real total returns in Adelaide and Sydney is even zero. The higher correlations in Finland are mainly due to the smaller geographical size of Finland: smaller size means more interdepence between the areas. That is the main factor for a Finnish real estate investor to seek for diversification opportunities to foreign property.

Table 2. Correlations of the real housing property returns in the sample areas

	Sydney	*Melbourne*	*Adelaide*	*Perth*	*California*	*Texas*
Sydney	1.000					
Melbourne	.652	1.000				
Adelaide	-.008	.433	1.000			
Perth	.728	.603	.349	1.000		
California	.336	.521	.059	.055	1.000	
Texas	-.324	-.130	-.010	-.367	.157	1.000
Florida	.268	.443	.012	.245	.534	.239
Virginia	.277	.449	.052	.122	.685	.190
Helsinki	.613	.663	.042	.450	.675	-.032
Turku	.726	.686	.092	.575	.663	-.131
Tampere	.664	.730	.038	.468	.701	-.075
Oulu	.636	.716	.090	.415	.703	-.071
	Florida	*Virginia*	*Helsinki*	*Turku*	*Tampere*	*Oulu*
Florida	1.000					
Virginia	.809	1.000				
Helsinki	.632	.622	1.000			
Turku	.517	.528	.940	1.000		
Tampere	.597	.577	.949	.913	1.000	
Oulu	.441	.478	.905	.914	.919	1.000

As expected the correlation coefficients between the total real returns in Finnish cities and Australian or American areas are much smaller than between the cities inside Finland. This is because the underlying factors do not correlate as closely between Finland and Australia or the United States as between different areas in Finland. However, there are some high correlations between cities in Finland and Australia. Adelaide is the only city in Australia, whose housing property returns do not seem to correlate with Finnish cities' returns at all. Surprisingly the correlations between Australian and Finnish cities are higher than the correlations between Australian and US areas. Although the GDP changes in Australia and Finland have been quite close to each other the growth rates do not explain this finding, Australian GDP changes have been in even slightly closer relationship to the growth rate of the US.

Observing that the Finnish economy follows relatively closely to the behaviour of the economy of the United States one can expect that correlation between housing property returns in Finland and in the US might be quite high. That seems to be the fact when we look at the correlation coefficients between California, Florida and Virginia and the Finnish cities — the coefficients are mainly much larger than 0,5. California's correlations are even higher with Finnish areas than with other states within the country. Texas, however, is an exception (as Adelaide in Australia). All the correlations between Texas and the Finnish centres are less than zero in the sample period.

So far we have not taken exchange rate fluctuations into account. Previously we have noted that standard deviations of real housing property returns are smaller in Australian and American areas than in Finland. The standard deviations of the returns[2] are however much higher in Australia and the United States than in Finland when measured in Finnish currency as displayed by figure 1. The returns have been counted as equally weighted averages for each country — each area has a weight 0.25.

[2] For now on Finnish inflation is used in computing the real returns of each return series.

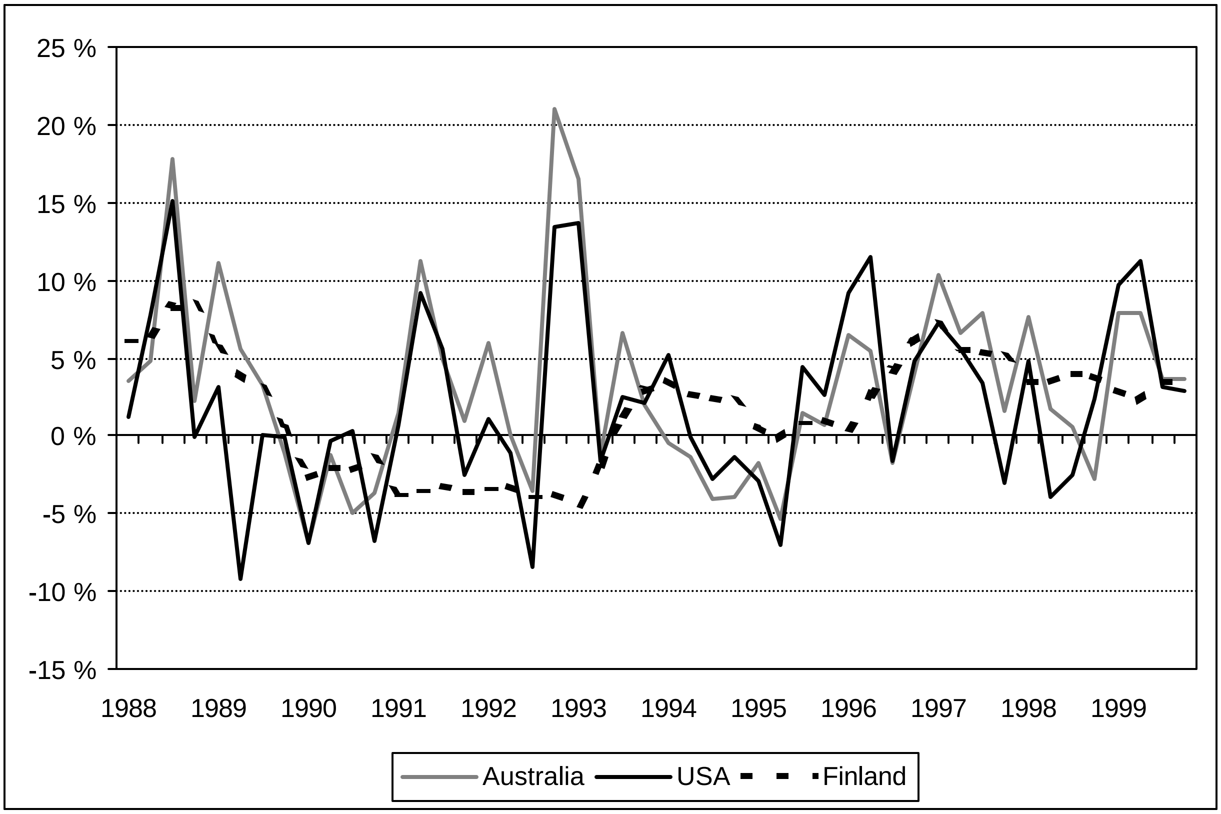

Figure 1. The quarterly real housing property returns expressed in the Finnish currency

The Australian and US standard deviations are large when measured in the Finnish currency because the exchange rate changes are taken into account. The standard deviation of the Australian real total housing property return is 6.00% (instead of 2.08% in local currency) and the US figure is 5.83% (instead of 1.51% in local currency). Furthermore the correlation between the returns in Finnish cities and the other areas are now significantly lower than in table 1. This shows the importance of exchange rate movements in international property investments: an investor should always take possible currency fluctuations into account. Figures 2 and 3 emphasize the relationship between the rate of return and the exchange rate movements.

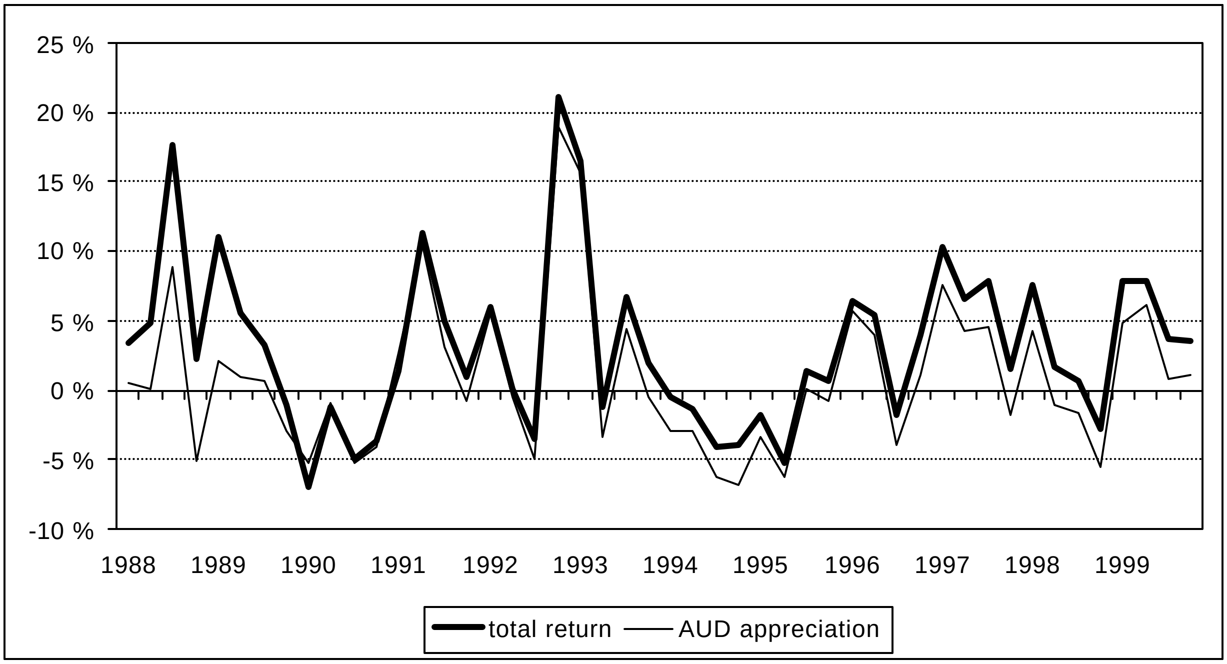

Figure 2. The quarterly real housing property returns in Australian cities (equally weighted) and the change in the value of the Australian dollar in Finnish Marks

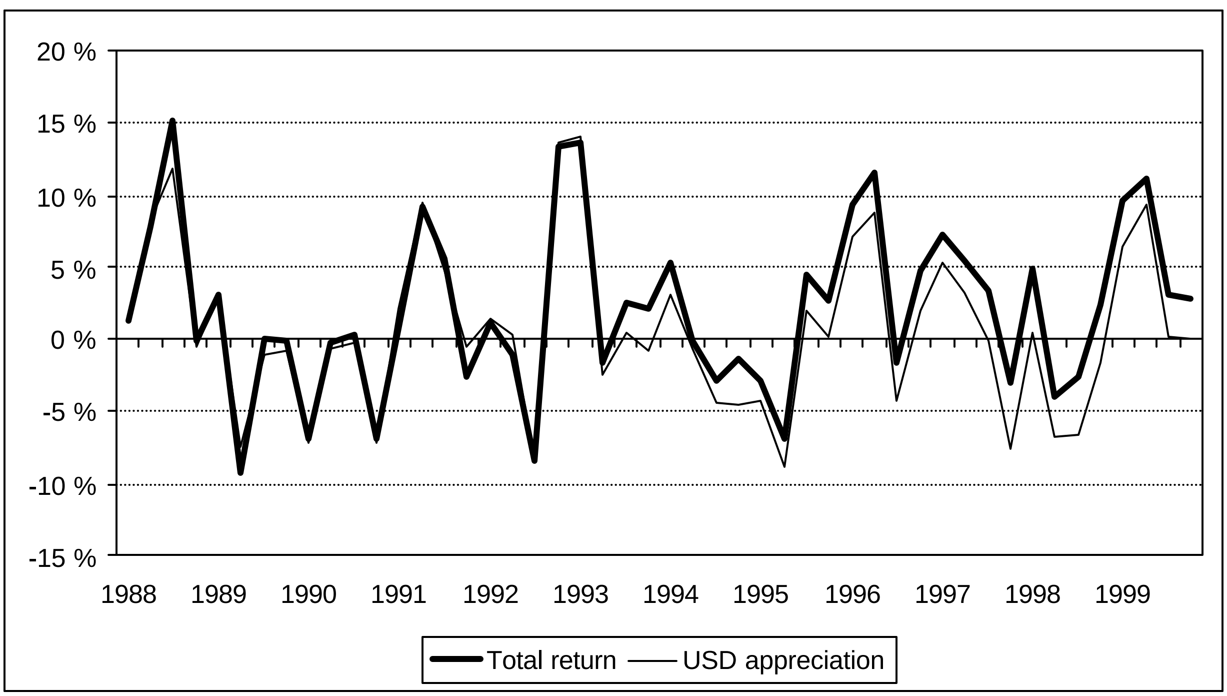

Figure 3. The quarterly real housing property returns in US areas (equally weighted) and the change in the value of the US dollar in Finnish Marks

As we can see the real returns follow the exchange rate movements very closely. The correlation coefficient between the real rate of return and the exchange rate changes is 0.94 in Australia and 0.96 in the USA. That means that when measured in the Finnish currency about 88 percent of the Australian and about 92 percent of the US real housing portfolio return movements can be explained by the changes in exchange rates. Furthermore, figures 2 and 3 show that the exchange rate fluctuations have been relatively large and this in turn has increased substantially the standard deviations of Australian and American housing property returns measured in the Finnish currency. This exchange rate effect is naturally similar in other investments too, but is it as significant as in property investment, is another question. According to Newell and Webb (1996) the significance of the exchange rate effect is larger for real estate than for bonds and especially for stocks.

The fact that makes housing property in Australia interesting to a Finnish investor is that in all the four cities in Australia the average quarterly real returns have been higher than in any of the Finnish centres when measured in the Finnish currency. Also in California, the total return has been significantly higher than in the Finnish cities during the sample period. The geometrical average real total returns in the Finnish currency and the standard deviations of the sample areas are presented in figure 4.

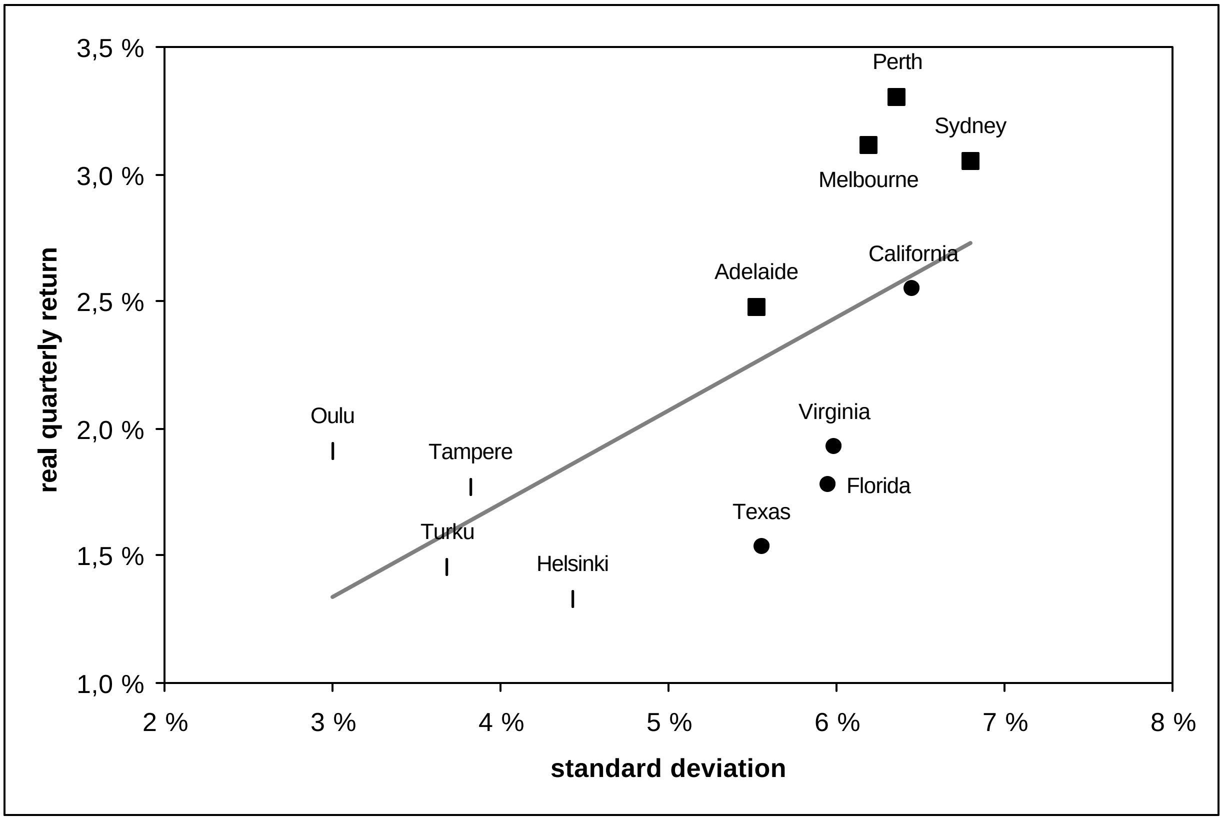

Figure 4. The average real total quarterly returns and standard deviations on housing properties in sample areas with the OLS fit

We are able to find some explanations for the downward sloping curve in the Finnish cities. First of all it is probable that the required rate of return is smaller in Helsinki than in the other three centres due to the better liquidity of housing property in Helsinki. The good performance of housing property in Oulu can be explained mainly by the immigration in Finland. All the four cities have been major Finnish growth centres during the sample period. However, the growth rate in Oulu's population has been a big surprise if we think about the expectations in the late 1980's. This can be explained mainly by the "Nokia effect". In Turku the development has been the quite opposite, population growth and immigration numbers have been relatively low lately.

The fact that the correlations between the returns of the Finnish and Australian cities are very low when measured in the Finnish currency makes Australian housing properties even more attractive to Finnish investors. The correlation coefficients between the sample areas are shown in table 3. Coefficients between the Finnish and Australian/US areas are bolded.

Table 3. Correlation coefficients between the real quarterly returns on housing property in four Finnish and Australian cities in the Finnish currency

	Sydney	*1.1.1 Mel-bourne*	*Adelaide*	*Perth*	*California*	*Texas*
Sydney	1.000					
Melbourne	.922	1.000				
Adelaide	.838	.942	1.000			
Perth	.930	.919	.886	1.000		
California	.783	.797	.709	.679	1.000	
Texas	.678	.718	.713	.630	.869	1.000
Florida	.789	.802	.767	.748	.920	.950
Virginia	.769	.781	.723	.701	.943	.917
Helsinki	**.235**	**.156**	**-.091**	**.135**	**.295**	**-.027**
Turku	**.288**	**.158**	**-.086**	**.191**	**.274**	**-.078**
Tampere	**.293**	**.215**	**-.054**	**.178**	**.316**	**-.032**
Oulu	**.251**	**.178**	**-.078**	**.120**	**.333**	**-.013**
	Florida	*Virginia*	*Helsinki*	*Turku*	*Tampere*	*Oulu*
Florida	1.000					
Virginia	.977	1.000				
Helsinki	**.102**	**.188**	1.000			
Turku	**.062**	**.140**	.940	1.000		
Tampere	**.105**	**.182**	.949	.913	1.000	
Oulu	**.091**	**.166**	.905	.914	.919	1.000

Adelaide and Texas have small correlation with Finnish cities in table 1 but now also the other bolded correlations are small although Sydney and California have slightly bigger correlations with the Finnish centres. The low correlations mean that Australian and American housing property will have a very good diversification effect in a Finnish housing property portfolio. As the exchange rate fluctuations have disappeared between the countries within the EMU, the significance of these results increase — the exchange rate effect that has lowered the correlations also between the housing property returns in Finland and the other member countries is now gone. This indicates that Finnish housing property investors have better diversification opportunities in countries that do not belong to the EMU, such as Australia and The United States.

Table 3 also shows that when measured in the Finnish currency the areal differences in Australia and the USA disappear. Texas has now high correlations with the other states and Adelaide with the other Australian cities. Also, the other correlations inside a country are higher. This causes that Finnish investors are not able to benefit from the areal differences inside Australia or inside the USA. The same effect applies to the correlations between Australia and the USA too — for example Texas has now relatively high correlation coefficients with Australian cities even though all the correlations are negative when exchange rate fluctuations are not taken into account. This has happened because the Australian dollar correlates strongly with the US dollar.

The results mean that as expected the diversification effect of adding Australian and/or American housing property into a Finnish housing property portfolio is significant. But the main source of the differences in the real returns is not based on the real estate specific factors — the low correlations are mainly due to the exchange rate fluctuations.

As noted before the results show that geographical diversification is not possible in Finland because the correlation coefficients between the cities are very high, even the smallest coefficient between Turku and Oulu is 0.905. Figure 5 shows the relationship between the real housing property returns in Finnish and Australian cities. The gray continuous curves present the returns in Finnish the cities and the black dashed curves present the returns in the Australian cities.

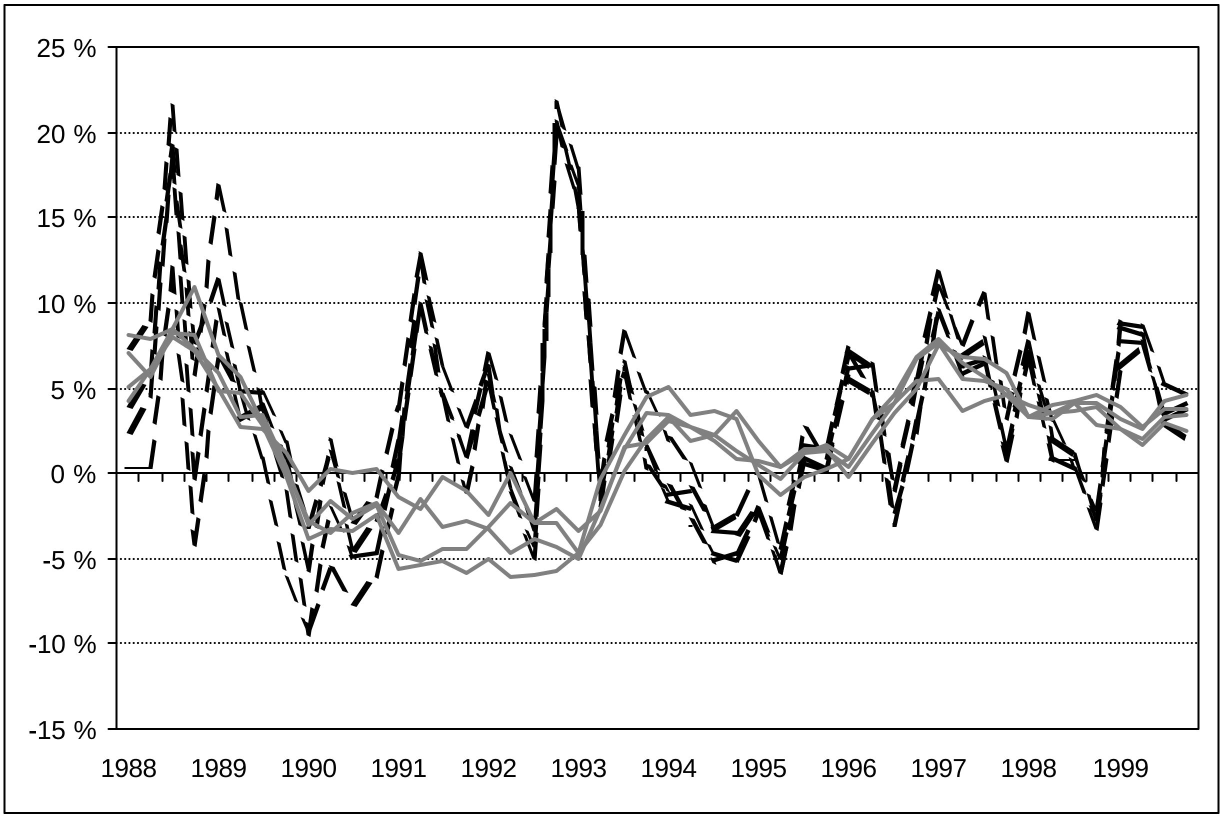

Figure 5. The real quarterly returns in the Finnish and Australian cities in the Finnish currency

As can be seen, the real quarterly returns in cities within both countries follow each other closely. The figure would be similar if it showed the returns in the USA instead of Australia.

The benefit that a Finnish investor can obtain by introducing Australian and/or US housing property into his housing property portfolio can be pointed out by showing how significantly the mean-variance opportunity set moves because of this action. Figure 6 presents the opportunity set that the Finnish investor faces if he invests in a portfolio that includes only the four Finnish cities.

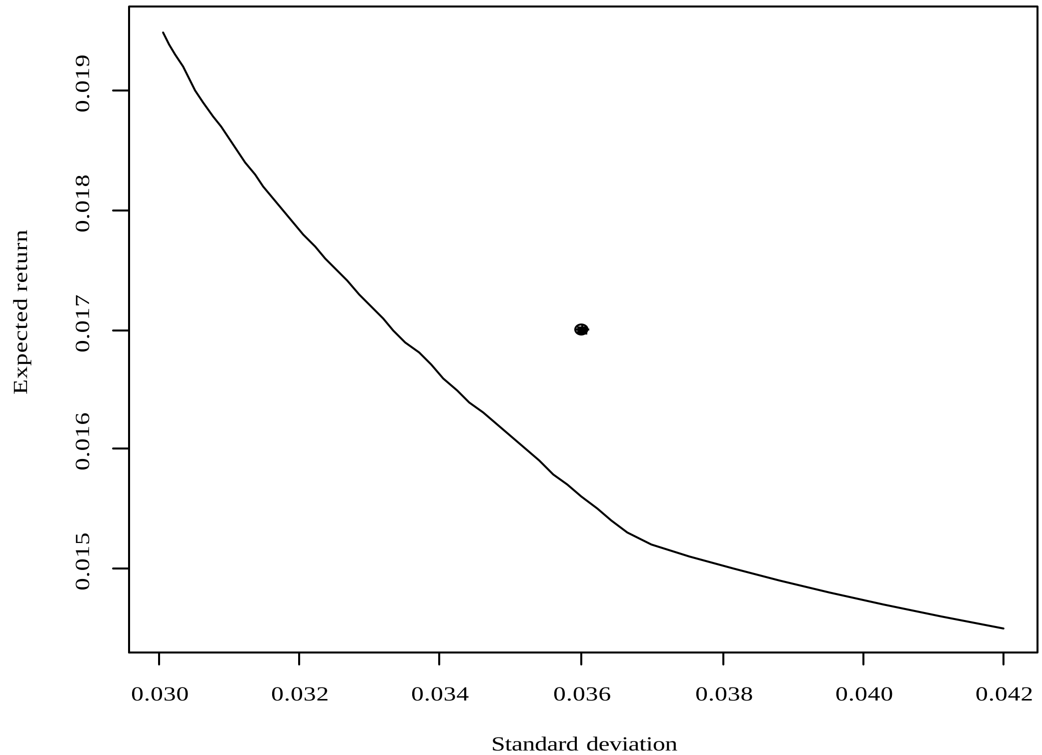

Figure 6. The mean-variance opportunity set[3] on Finnish housing property portfolio

The reason for the strange shape of the mean-variance opportunity set can be explained by figure 4. In the four Finnish cities the risk-return coefficient has been negative. This together with the limitation that short selling is not possible leads to the shape we observe in figure 6. The black circle in the figure is the equally weighted portfolio which is far away from the efficient frontier. The minimum risk portfolio, with quarterly expected return 0.0195 and standard deviation 0.0300) consists of the housing property in Oulu and Turku with weights 0.981 and 0.19 respectively. Because of the shape of the efficient set this minimum variance dominates all the other possible portfolios.

Figure presents the opportunity set that Finnish investors faced if they invested both in Finnish and Australian housing properties. By investing in both countries' housing properties a Finnish investor could have the same expected return with a lower level of risk or significantly higher expected return with the same level of risk. Furthermore, the investor would have much wider range of possible expected returns to choose. The minimum vari-

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[3] The expected returns and standard deviations are based on quarterly figures. Furthermore, the figures are based on the historical returns and variances during the sample period.

ance portfolio[4] has the expected return of 0.0212 and standard deviation 0.0254 in this case. Again, the black circle displays the equally weighted portfolio which would not be efficient.

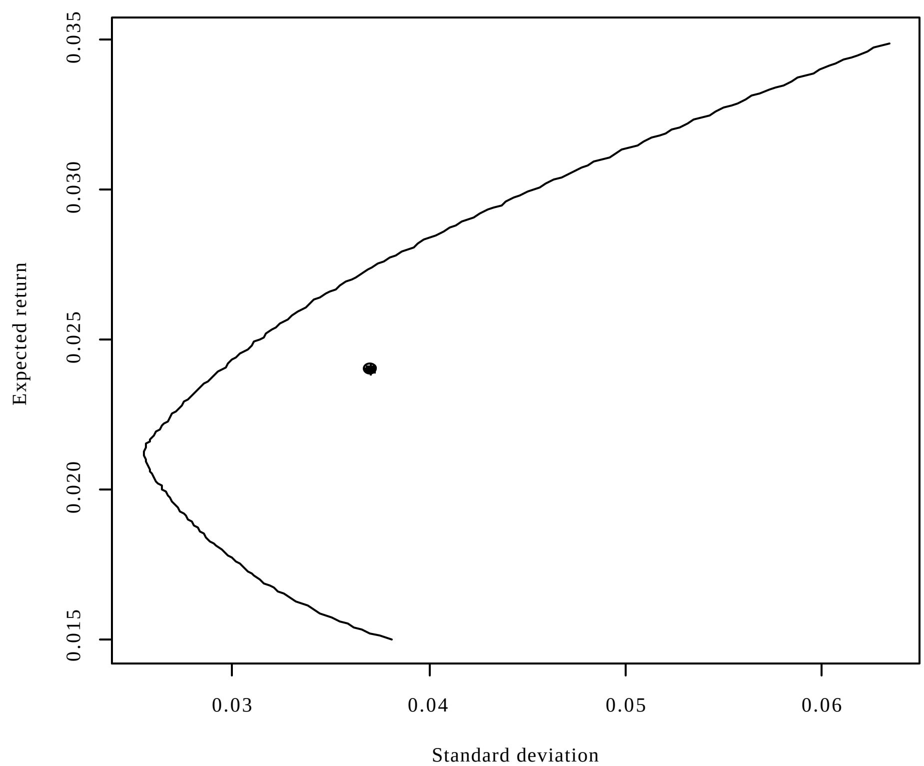

Figure 7. The mean-variance opportunity set on Finnish-Australian housing property portfolio

As we go on diversifying the Finnish-Australian portfolio by including also US housing, property the efficient set naturally goes further up — we get better expected return at the same level of risk. The minimum risk portfolio consists now of Oulu, Adelaide and Texas with weights 0.737, 0.174 and 0.088 respectively. The fact that there is only one area from each country in the minimum variance portfolio is due to the high correlation between the areas within each country. The minimum variance portfolio has expected return of 0.0205 and standard deviation of 0.0252

[4] The weights are 0.756 for Oulu and 0.244 for Adelaide.

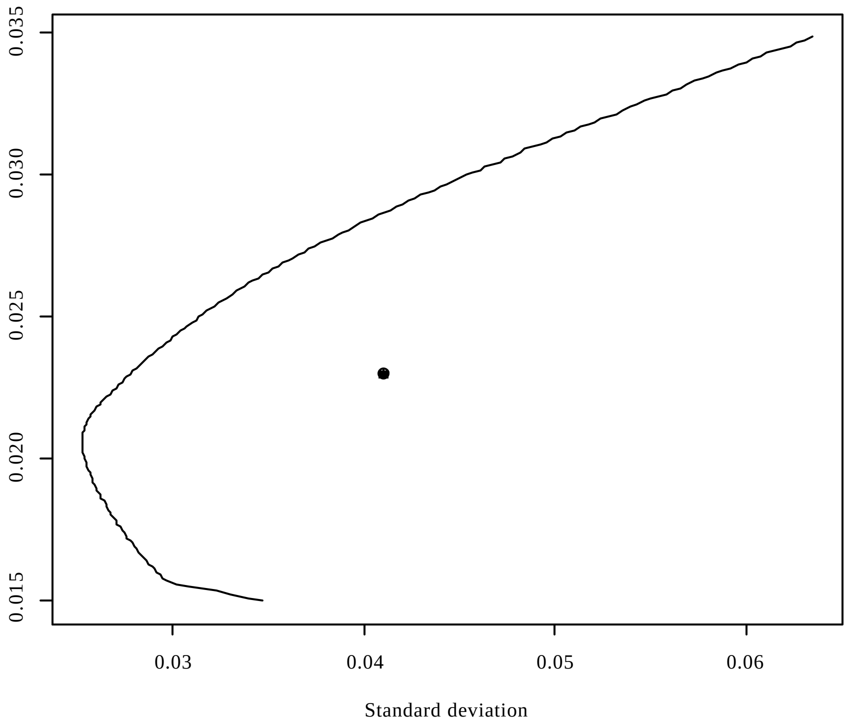

Figure 8. The mean-variance opportunity set on the portfolio that consists of Finnish, Australian and US housing property

5. Conclusions

The purpose of the paper was to study if a Finnish housing property investor is able to diversify by investing in Australian and US housing property. The formal analysis was based on modern portfolio theory. Quarterly return data from 1988 to 1999 of four Finnish and four Australian cities and four US states was used to study the hypothesis.

As may be expected, the empirical results showed that correlation coefficients between US, Australian and Finnish cities were relatively small. Because of this, introducing Australian or US housing property into a Finnish housing property portfolio moves the mean-variance opportunity set substantially. The analysis also showed that exchange rate variability is a major factor influencing the Australian and US housing property returns the Finnish currency.

It should be noticed that the analysis is based on historical data. The short sample period may not give reliable information on expected returns. The returns differ from the mean blur, such as the same in the future is unsure. Furthermore, it will cost for a Finnish investor to collect the necessary information on Australian and US housing property markets. However, the diversification benefit of adding Australian and/or US housing property seems to be significant, so that the information gathering costs would not make the international diversification action unprofitable. Furthermore, the standard deviations of the portfolio returns in the minimum variance portfolios are larger than the expected returns in every case. In this analysis we have ignored the institutional differences between real estate investment in Australia, Finland and USA. Also some critics may be presented about the adaptability of mean-variance analysis into real estate investments.

Appendix

Consider N financial assets and denote $\tilde{R}_i$ is the mean return on asset i. Let $\tilde{R}$ be the vector of mean returns:

$$\tilde{R}^T = \left[\tilde{R}_1, \ldots, \tilde{R}_N \right]^T .$$

The task is to find vector α to represent the proportional weights of the assets in the portfolio. The investor's decision problem is

$$\max_{a} \Phi E\left[\tilde{R}_p \right] - \boldsymbol{s}^2 \left[\tilde{R}_p \right]$$
$$w.r.t. \begin{cases} \sum_{i=1}^{N} \boldsymbol{a}_i = 1 \\ \boldsymbol{a}_i \geq 0, \end{cases}$$

where Φ denotes the coefficients of required returns, $E\left[\tilde{R}_p \right]$ is the vector of expected returns such that $\Phi\ E\left[\tilde{R}_p \right]$ is the required return on the portfolio and $\boldsymbol{s}\left[\tilde{R}_p \right]$ is the vector of the standard deviations of the returns. The returns are normalized to have $\tilde{R} \sim N(\boldsymbol{m}, V)$ such that

$$E\left[\tilde{R}_p \right] = \boldsymbol{a}^T \boldsymbol{m}$$

and

$$\boldsymbol{s}^2 \left[\tilde{R}_p \right] = \boldsymbol{a}^T V \boldsymbol{a} .$$

The problem is solved by the result of a standard quadratic programming problem

$$
\begin{aligned}
&\min_x \frac{1}{2} x^T Q x - x^T R \\
&w.r.t. \begin{cases} Ax = B \\ Cx \geq D \\ e_1 \leq x \leq e_2. \end{cases}
\end{aligned}
$$

In this case the portfolio problem is represented by

$$
\begin{aligned}
&\min_{\boldsymbol{a}} \frac{1}{2} \boldsymbol{a}^T (2V) \boldsymbol{a} - \boldsymbol{a}^T (\Phi \boldsymbol{m}) \\
&w.r.t. \begin{cases} 1^T \boldsymbol{a} = 0 \\ I \boldsymbol{a} \geq 0. \end{cases}
\end{aligned}
$$

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