

UNCERTAINTY, FLEXIBILITY, VALUATION AND DESIGN: HOW 21st CENTURY INFORMATION AND KNOWLEDGE CAN IMPROVE 21st CENTURY URBAN DEVELOPMENT – PART I OF II

DAVID GELTNER AND RICHARD DE NEUFVILLE
Massachusetts Institute of Technology

ABSTRACT

The 21st century presents humankind with perhaps its greatest challenge since our species almost went extinct some 70,000 years ago in Africa. A big part of meeting that challenge lies in how the urbanization of three billion additional people (equal to the entire world population in 1960) will be accomplished between now and mid-century, on top of necessary renewal and renovation of the earth's existing cities. China alone will urbanize 300 million more people between now and 2030, equal to the entire population of the U.S., the world's third most populous country, in just 20 years. This is development on a scale and pace that is an order of magnitude greater than the past century, in a world resource and climate environment that is near the breaking point, in a context of greater technological, financial, and economic uncertainty than ever before.

To meet this challenge will require that we use the best tools in our kit, including ones that have become available to us only in this new knowledge and information-based century. Technology got us here, and technology will be key to getting us through. In this paper we will review and synthesize two important methodological developments in our profession that can help infrastructure and real estate physical development (i.e., urban development) to be accomplished more effectively and efficiently in a world of uncertainty.

The first methodological development is the honing of real options theory and methodology for practical application to identify and evaluate sources of flexibility in the design and operation of capital projects. The second development is the marriage of digital data compilation of property transactions records with the honing of econometric analysis methodology to allow the practical quantification of real estate and infrastructure asset price dynamics.

We argue that this latter development provides the key input to the former development, enabling a much more complete and rigorous treatment of design and evaluation problems for urban development. We also argue that an engineering systems approach to option modelling is likely to find better traction in actual professional practice than the economic theoretical models that have dominated the academic literature. We provide a concrete example by applying the suggested approach to the Songdo New City development in Korea.

The result can be better informed design and valuation and more efficient urban development laced with greater flexibility to avoid the worst down-side outcomes and to take advantage of the best up-side opportunities, saving vital resources of capital, land, raw materials, and energy.

Editor's Note: Professors Geltner and de Neufville's substantial paper has been divided into two parts. The first part (Vol 18, No 3, pps 231-249) includes the introduction and consideration of economic real option models, engineering models and Monte Carlo simulation. The second part (Vol 18, No 3, pps 251-276) includes a consideration of quantifying uncertainty or volatility through real asset pricing data and indexing, an application to Songdo New City and conclusions. For ease of reference, the abstract and references have been included in both parts.

Keywords: real options, design, digital, uncertainty, development, Songdo New City

INTRODUCTION

The 21st century presents humankind with perhaps its greatest challenge since our species may have approached extinction in Africa some 70,000 years ago.¹¹ Population will likely peak by the middle of this century at around nine billion, almost a quarter more than today and three times what it was when the co-authors of this paper first entered MIT as undergraduates half a century ago. For the first time in history, half the human race now lives in cities, and while the world rural population is already essentially at its peak, urban population will double, adding another three billion people to cities by the middle of the century. In China alone, in just the next 20 years, 300 million people will become urbanized, equal to the entire (not just urban) population of the United States, the world's third most populous country. Over just the next 10 years The Economist Intelligence Unit predicts China will invest over USD 11 trillion on urban housing alone (almost the magnitude of the entire current U.S. annual GDP). This is development on a scale and at a pace that is an order of magnitude greater than in the past century, which was itself already earth-shaking in many ways. Development in the first half of the 21st century will require massive investment in infrastructure, housing, and commercial real estate. And this development must occur in a world where resources are constrained as never before.

Furthermore, it is now clear that climate change will be an increasing factor throughout this century. Development must occur in the context of a need to transform production and consumption patterns and technology to prevent further damage to the environment not least being a possibly catastrophic escalation of global warming. This will require a revolution in energy sources and usage. Against these challenges, humanity is now armed with an incredible level and growth in technology and information, and in global productivity and wealth, as well as supra-national institutions, far beyond what previous centuries have had at their disposal. The challenge is unprecedented, but so is our means to address it. They say that "change is the only constant", but what we are facing in the coming decades is not just revolutionary change, but a degree of uncertainty that is perhaps unprecedented in human history. If you think the first decade of the 21st century was volatile, our guess is, "*you ain't seen nothin' yet!*"

What is the meaning of this challenge for we who are professionals with important responsibility in the planning and design, construction, and financing of the infrastructure and real estate that will define the built environment of the 21st century? This question may have many answers, but among them are likely to be words such as "urgency" and "humility". Urgency because we are literally in a race between the unleashed explosive forces and effects of rampant development on the one hand and the advance of rational controlling and guiding powers that will be necessary to shape these forces for the good on the other hand. Humility because we cannot possibly know what the future will bring, and we have made major mistakes in the past. Infrastructure and real estate are huge fixed investments. (For our purposes in this paper we will define "infrastructure", like real estate, to refer to long-lived, capital-intensive, spatially-fixed real assets, whether in the private or public sector.) The quantity, quality, type, and location of such real asset placements in the 20th century have not always proved to have been optimal or wise, in retrospect, even when they seemed so logical at the time when they were made.¹² The as yet unknowns that await us in the 21st century loom even larger than those that humbled the planners and builders of the last century.¹³

¹¹ Genetic evidence suggests that our ancestors may have been reduced at that time to perhaps barely over 1,000 breeding pairs, a level perilously close to the point at which extinction would almost surely have resulted. It is not known exactly why or how this "bottleneck" in human evolution occurred, but a result is that at the genetic level our species exhibits much less diversity than most species.

¹² Many of the massive "slum clearance" so-called "urban renewal" projects undertaken in the United States in the 1950s and '60s are an obvious example that comes to mind. To go back even earlier, it would seem that the U.S. built about twice as much railroad mileage as it soon actually needed, as over 100,000 miles of track have been abandoned beginning in the early to mid-20th century, more abandoned than the entire existing railroad mileage in all but one other

With this in mind, it is the thesis of this paper that the design and development of the built environment in this century must be done with a greater understanding and appreciation of the value of, and need for, *flexibility*, than has heretofore been the case. Flexibility, and particularly as implemented by the explicit inclusion of options, or “optionality”, within major development projects, can be a major rational response to the great uncertainty and change that this century will certainly witness. Flexibility in design and decision-making can enable more efficient and effective use of scarce resources. It can enable developers and users of the built environment to take advantage of unexpected upside opportunities while also facilitating the avoidance of untoward downside events.

More specifically, we write this paper with the intent of weaving together three strands or themes that have arisen in the past couple decades in both the academic literature and professional practice of the engineering, design, and financial communities that focus on real estate and infrastructure development. We believe that, applied synergistically in an integrated fashion, these three strands can raise the professional level of practice in real estate and infrastructure design and investment so as to take advantage of opportunities for flexibility. The three strands we are referring to are:

1. **Real Options Models:** the development in the financial and real estate academic literature of sophisticated, rigorous economic models of the valuation and optimal exercise of “options”, that is, the right without obligation to undertake (or delay or abandon) a physical capital investment, such as buildings or highways, in an environment of uncertainty;
2. **Monte Carlo Simulation:** the development and honing of practical engineering analysis tools to model and value decision flexibility in the design and operation of physical assets in a context of uncertainty, in a manner that can be effectively communicated to and used by professional decision makers; and
3. **Real Asset Pricing Data and Indexing:** the development of vast electronic databases of housing and commercial property asset transaction prices and appraised valuations which, combined with advances in econometric methodology and computation (and supplemented by some evidence regarding privatized infrastructure investment performance), allow much more complete and rigorous quantification of the aforementioned economic and engineering models applied to real estate and infrastructure development projects.

We will present a brief review of each of these three developments, and then we aim to show how the three are mutually reinforcing and synergistic, with the economic real options model providing important theoretical underpinning to a type of Monte Carlo simulation model that can be more useful than the economic model in design practice, and with the asset pricing data and indexing advances resolving a key and fundamental “GIGO” (*garbage in, garbage out*) input problem for both types of models by enabling better quantification of the relevant volatility and uncertainty. We will apply these tools to a stylized but realistic example of a major development project that may be prototypical of much of the new urbanization of the 21st century in the emerging market countries, that of the Songdo International Business District development in the Republic of Korea.

country (Russia). (Of course this does not *necessarily* imply that all of that abandoned track should never have been developed in the first place.)

¹³ See Macomber (2011) for an important perspective on the big picture of how and where capital can interact with design to intervene most effectively in the great global urbanization process that is challenging humanity.

The paper is organized into five sections. We begin with sections reviewing each of the three aforementioned methodological developments as they may be applied to real estate and infrastructure development. A fourth section then synthesizes the three and applies them to the Songdo example. A fifth section provides our concluding remarks.

ECONOMIC REAL OPTIONS MODELS

The concept of “options” is most traditional in the field of finance, where “call options” and “put options” on stocks have been traded for decades. Fundamentally, an option is a “*right without obligation*”, for example, a *call* option gives its owner the right to buy a specified stock at a specified price, but the option owner does not have to exercise that right. The object to which the right applies is called the *underlying asset* of the option. A *put* option gives the right to sell its underlying asset at a specified price.

A *real option* is an option in which the underlying asset is a physical asset or collection of physical assets, such as mines or drilling platforms, factories, commercial buildings, or infrastructure facilities such as highways or ports. In the sorts of real options that are the focus of the current paper, the exercise of the call option on the asset is associated with the construction of the asset. The *exercise price* (or “strike price”) of the option is the construction cost of the project (exclusive of land cost). If the ownership of an undeveloped parcel of land conveys the real option rights to develop the land, then the value of that option is the value of the land parcel (and vice versa, in essence, the value of the land is the value of the real option). The value of the vacant land is completely based on the nature and timing of the development decision that can (but need not, if it is an “option”) occur for the land.

The economics-based real option model of land value and optimal development arose from two strands of the academic literature. In economics departments and finance departments in business schools, mathematical models of financial options were developed in the late 1960s and 1970s, most famously with the 1972 Black-Scholes Model of so-called European calls and puts (options that can only be exercised on their expiration dates), for which the Nobel Prize in economics was awarded to Myron Scholes and Robert Merton in 1997. In a famous 1977 article MIT professor Stewart Myers coined the term “real options” when he extended the financial call option model to the value of “growth opportunities” a corporation holds in the investment projects they could implement (at their discretion). By the 1980s the real options concept was being applied to capital budgeting and project development investment decisions by authors such as McDonald and Siegel, with an influential book published by Dixit and Pindyck in 1994. A key focus was on the “value of waiting” to invest, and the need for the project developer to see more than just a zero net present value (NPV) comparing the construction cost of the project to the anticipated value of its benefit. In essence, options have value because of their non-obligatory nature. They enable the option holder to take advantage of upside possibilities that arise in the future, while not forcing the option holder to enter into an unfavourable outcome when downside eventualities occur.¹⁴ This makes options a bit of an oddity in the financial world: assets whose values are *increased* with greater volatility (i.e., with greater risk). Greater volatility means a greater downside possibility, but also a greater upside. An option allows one to take advantage of the upside without exposure to the downside. Hence, volatility is a “friend” to an option holder.

¹⁴ An analogy has been made using a surfing metaphor. The surfer paddles out beyond the breakers and waits. He doesn't take the first swell that comes along (usually), because there may be a better wave beyond that one, or the next. This is analogous to not building a development just as soon as its value exceeds only its construction cost. But the surfer doesn't wait forever either. A good enough swell will make it worthwhile to give up the opportunity to keep waiting for an even better wave. The ability to wait and select the wave of his choice is a big part of what makes the surfing fun, what gives the surfing its “value”. This is analogous to the option value.

The second strand of the real options literature came out of urban economics and real estate, with the publication in 1984 by Sheridan Titman of “Urban Land Prices Under Uncertainty”, the first explicit presentation of the call option model of land value as described above, showing that uncertainty in the real estate market increased land value but delayed land development. Dennis Capozza and Robert Helsley extended the classical “monocentric city model” to include the consideration of uncertainty, a fusion of spatial and financial economics that showed that optimal development under uncertainty would produce a denser and higher-rent city. Joe Williams and others extended the model to consider optimal development density as well as timing, and there has been a thriving real options literature in the real estate and urban economics fields, as well as in capital budgeting and corporate finance, ever since the 1980s and continuing into the 2000s. More recently there has been a focus on empirical verification of the theoretical real option model, and several careful studies have generally confirmed the empirical validity of the real option approach as a model of land value and development.¹⁵

The key insight from the economic real option model for purposes of the present paper is that *flexibility contains value* in the context of physical capital investments, and this value is *greater* the *more uncertainty* or volatility exists in the value of the underlying assets being developed. In essence, an option (or more generally, “optionality” in development projects) provides flexibility. Options allow the developer to build later (or sooner), to build more (or less) on a given site, or to build different types of structures. From a systems perspective, it is sometimes useful to view these types of flexibility as providing options *on* the system. (This is in distinction from options *in* the system, which have to do with a more micro-level of design and operational decision-making, where the economic model may be less useful.) The real options literature proposes a number of ways in which major capital projects often contain flexibility that can be evaluated as options. For example, Trigeorgis (1996) enumerates the following taxonomy:

- Option to Defer
- Time-to-Build Option (Staged Investment)
- Option to Alter Operating Scale (Expansion, Contraction)
- Option to Abandon
- Option to Switch (Outputs or Inputs)
- Growth Options
- Multiple Interacting Options

The importance of the economic model’s insight about the value of flexibility cannot be overstated. But the practical importance of the economic real option model is not just in this insight, but also in its ability to *quantify* the value of flexibility in development projects, at least at an approximate and broad-brush or large-scale level, in a very *rigorous* manner, based on economic science. To see this, consider the theoretical basis of the economic options model.

It is often believed that the theoretical underpinnings of the economic option model make it only valid or relevant in circumstances where the underlying assets are traded in “perfect markets”, that

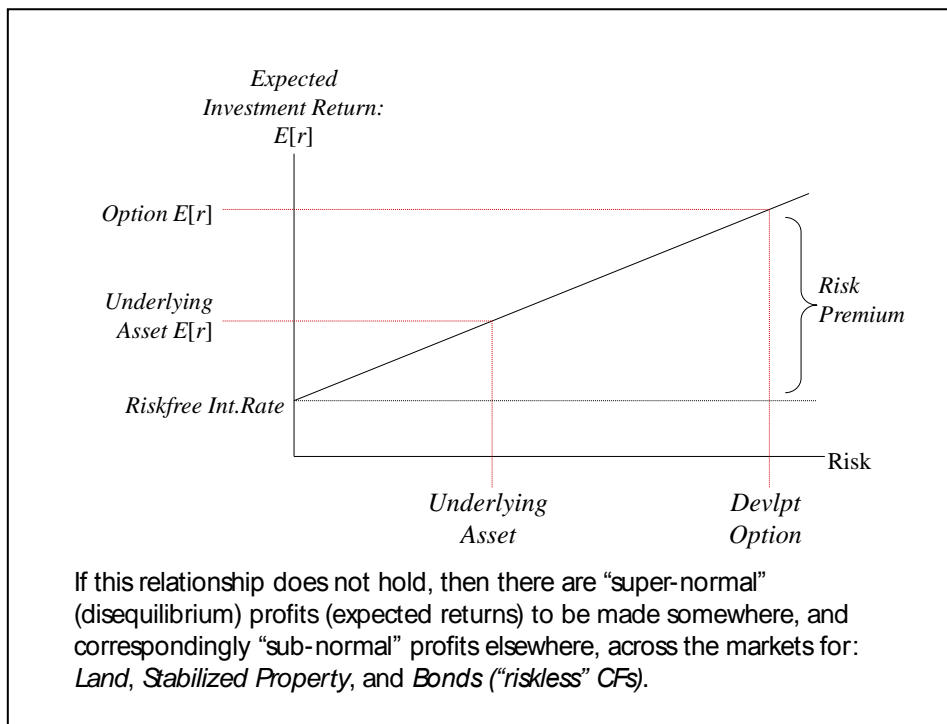
¹⁵ See for example recent articles by Cunningham (2006, 2007), Schwartz and Torous (2007), Clapp and Lindenthal (2009), and Bulan *et al* (2009), which built on much earlier work by Quigg (1993). Also worthy of note was a series of articles by Steve Grenadier applying the real option model to optimal leasing and to help explain the over-building phenomenon or tendency toward “development cascades” in real estate. Indeed, there are by now literally hundreds of academic articles applying real option theory to real estate and capital budgeting. And more recently a few articles have applied real options theory specifically to infrastructure projects, such as Chiara *et al* (2007), Rose (1998), and Smit and Trigeorgis (2009). All the references cited or mentioned in this paper are contained in the bibliography at the back, but we are not claiming this does more than highlight a few of the more famous articles, and we are leaving out many fine contributions.

is, highly competitive and frictionless markets where assets are homogeneous, can trade in any fractional shares long or short, all prices are immediately publicly quoted, and transactions are free and immediate. But in fact the theoretical underpinnings of the economic option model are much more broadly relevant than in such a rarified circumstance.

It is true that the original derivations of the option valuation model in the financial economic literature employed arbitrage analysis, in which the mathematical formulas were derived by imposing no-arbitrage conditions on the price the option could command. As an “arbitrage” involves the making of certain money risklessly, it is presumed that any such opportunities will be quickly seized and, in a competitive market, prices would adjust until no arbitrage is possible. The no-arbitrage condition is thus a powerful descriptor of equilibrium prices in the markets relevant for trading the option. But equilibrium pricing is a deeper and broader concept than the no-arbitrage condition. In that broader context one may view the no-arbitrage condition as merely a technical device for arriving at an equilibrium pricing model. If the relevant markets are so complete and frictionless as to enable true arbitrage trading to occur, then indeed such trading will “enforce” in reality the equilibrium price derived by the model. (In the real world no markets are quite as “perfect” as the mathematical models assume, but some come close enough to make arbitrage trading a major reality, such as some markets for stocks, bonds, commodities, foreign exchange, and various derivatives based on these underlying assets.)

But even in the absence of perfect conditions and arbitrage trading, markets still exist and often function well, and find (or tend toward) equilibrium between demand and supply. Thus, an equilibrium price model will be a “good” model even in less than perfect markets where literal arbitrage is not possible, such as real estate markets. The option pricing models derived using the no-arbitrage condition describe equilibrium pricing of the options, prices that balance supply and demand, whether or not such equilibrium is “enforced” by arbitrage trading. Indeed, such models in the finance literature have often also been derived using explicit equilibrium models such as the Capital Asset Pricing Model (CAPM). The way to look at the economic option price models in a context of well-functioning but imperfect markets such as real estate is that the models provide a good guide for what the option would likely sell for. While any given deal may deviate from the model’s prediction, the model will tend to predict the correct price on average.

In fact, we can go even farther. The economic option value models have an important type of validity in a context where the relevant market is not very good at all, or even non-existent (such as, perhaps, some very unique underlying assets or some types of infrastructure assets that are not traded in the private sector at all). In this context, there is no market value or exchange value for the option, but there may still be a type of “opportunity cost”. Resources will be expended to acquire and exercise the option, and it will produce an asset that has value. One may define the option value in such circumstances based on *normative* considerations, and view the valuation model from such a normative rather than *positive* (or empirical predictive) perspective. From this perspective the economic option value model may be viewed simply as implementing the “Law of One Price”, requiring that the option and its underlying asset and riskless bonds all provide expected returns on the same “Security Market Line.” That is, each investment must provide the *same* going-in expected investment return risk premium (above the risk-free interest rate) *per unit of risk*. This would seem to be quite a reasonable basis on which to define a normative definition of value for the option, in relation to its underlying asset and construction cost.



**The “Law of One Price” and the Security Market Line:
The Theoretical Underpinning of the Economic Option Value Model**

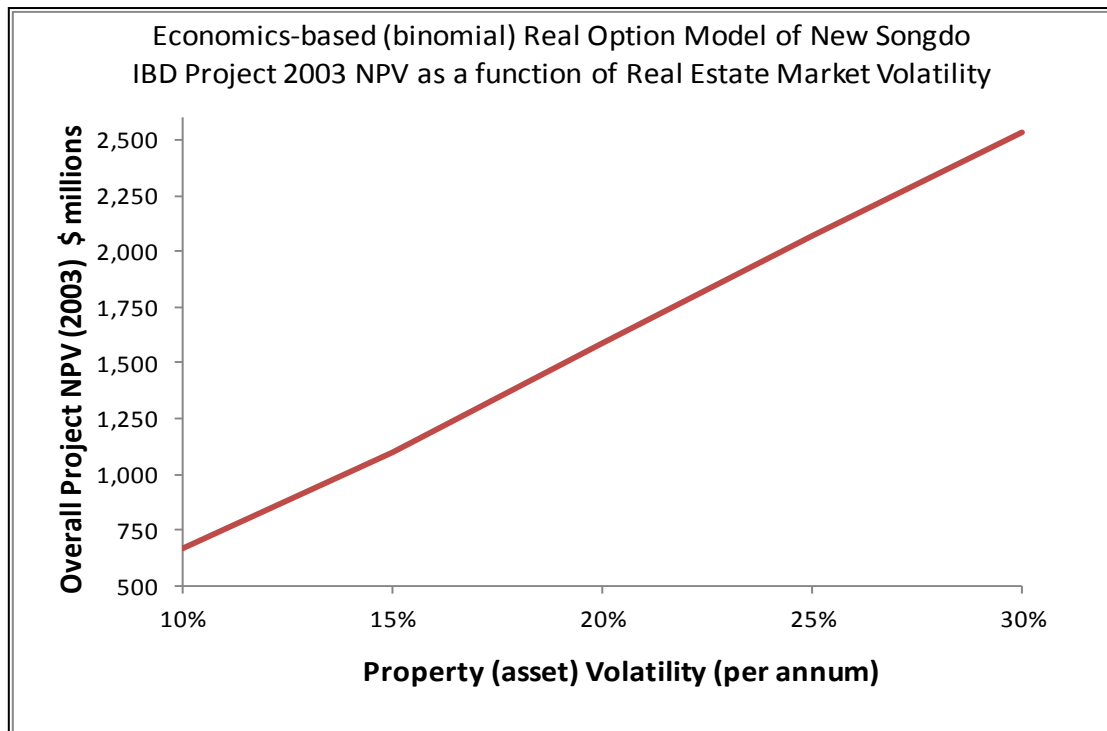
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Exhibit 1

This “Law of One Price” essence of the theoretical underpinnings of the economic option value model is depicted in Exhibit 1. The horizontal axis measures risk in whatever way the capital market cares about risk and thereby reflects risk in asset prices. The vertical axis measures expected investment returns. The straight line, a security market line (SML), indicates the expected returns that will provide all assets, including both underlying assets and derivatives such as options, with the same expected return risk premium per unit of risk. The economic option value model simply indicates the option price that will cause the option to provide such an investment return expectation. Any other price would cause the option to either provide more, or less, return premium per unit of risk than is “normal” (as indicated by equilibrium or “average” prices of assets in the capital market). If nothing else, such a “miss-pricing” would seem to be unfair in some sense (or to some party).

The real option valuation model is unquestionably one of the major discoveries in the economics of finance and real estate in the past generation. And yet, interestingly, it is very little used in the actual real world of professional practice, either in valuation or in project investment analysis and decision-making. This is in spite of the fact that it would seem, in principle, to be of considerable practical import in both of those fields. And it is also in spite of the fact that financial option models have achieved widespread practical use in the investment world. For example, stock options and derivatives are regularly valued and traded with the aid of option valuation models, and whole new indices and major investment products have been based on such models (for example, the “VIX” index tracking the volatility in the stock market). In our view, there appear to be two major reasons for the lack of practical usage of the real option model, at least regarding application to real estate and infrastructure development projects: difficulty quantifying the model’s required inputs, and the complexity or opacity of the models regarding practical decision making. We will attempt to make a contribution to both of these issues in the present paper.

The problem with the quantification of the inputs to the real option model is summarized by the classic expression: “*Beware of GIGO!*” (*Garbage in; Garbage out*). The economic real option model requires several quantitative inputs, but perhaps the most problematic for application to real estate and infrastructure projects has been the metric that the model uses to measure the relevant uncertainty regarding the underlying asset’s value. In the most basic and potentially widespread applications, this input represents the volatility in the underlying asset’s value, that is, the longitudinal standard deviation in the returns to an investment in the underlying asset. Option value can be extremely sensitive to this input. For example, drawing on our Songdo project example application that we will elaborate on in a later section, Exhibit 2 shows how development project values can vary several-fold within a reasonable range of input values for underlying asset volatility. For financial option model applications in the securities investment world of publicly-traded homogeneous assets, such as shares of stocks or commodities or foreign exchange, a wealth of data has long been available to quantify actual historical volatility and asset price dispersion, giving investors and traders confidence to use the option models. But until recently there has been no such similar empirical data on asset prices in the world of private markets trading unique whole assets, such as real estate. We will discuss the recent developments in this field further below.



Sensitivity of Project Value as a Function of Volatility of Assets to be Built

Source: Authors

Exhibit 2

The second problem underlying the lack of widespread usage of the real option model in professional practice relates to the nature of the model itself. The derivation of the model as an artefact of the discipline of economics is both a strength and a weakness regarding practical adoption. The strength is the aforementioned rigor of the model, the clarity and elegance of its theoretical underpinnings in the concept of the Law of One Price. But the model contains two weaknesses that undercut its widespread use. One is that its mathematics tends to appear complex and opaque to the uninitiated and non-specialized (ie: to most people who aren’t card-carrying economists). The mechanics of the model, in terms of formulae or algorithms, typically lack much

intuitive appeal for many potential users and decision makers. As a result, they have difficulty trusting or understanding the meaning of its prescriptions, and they tend to shy away from its use.

Another problem is that while the mechanics of the model may appear complex, the essential nature and assumptions of the model actually often make it overly simplistic or inexact in terms of representing specific real world design and investment decision choices and behaviours. The model tends to apply “*at 30,000 feet*”. This relates to our previous point about modelling “*options on*” as distinct from “*options in*” the building and infrastructure systems that compose the actual “bricks and mortar” of the development projects. The economic real option model may be good for evaluating the option to build a particular building, but it is less well suited to modelling the design question of whether to build in the structural capability to expand the building vertically from its initial 30 stories to a possible subsequent 50 stories without disrupting the occupants of the initial 30 stories, for example. It may be good for elucidating in principle how a developer should decide optimally to pull the trigger on a development (in terms of a critical or “hurdle” value of the asset to be built compared to its construction cost, for example), but it may not be good at modelling how the decision makers actually make such a decision, including consideration of available debt and joint-venture partner financing terms, likelihood of initial anchor tenant leasing, observable vacancy levels or lease-up rates in the local market, knowledge about specific competitive projects, and so forth.

This second problem in the use of the real options model derives from its very nature as an artefact of the economics discipline, and as a result, we suggest that to address this issue one must move beyond economics, to a fundamentally different type of model. We call this the “engineering model” of flexibility in development projects, and its major tool (in our current context) is the use of simple Monte Carlo simulation modelling in Excel®. This approach is admittedly less rigorous (for example, it cannot root its prescriptions in equilibrium theory or the Law of One Price), but it can better address the problems of opacity and over-simplicity for modelling specific decisions relating to flexibility not just “on” but also “in” development projects. It has obtained more widespread usage in actual practice, so far not much in real estate and infrastructure projects, but in manufacturing and natural resource extraction. Furthermore, we view the economics model and the engineering model as complements to each other. The economics model can be used to help calibrate and confirm the engineering model by applying them side-by-side to simple problems to which they can both be applied (such as the valuation of a basic “option on” a particular building project). It is to the engineering model that we turn in the next section.

ENGINEERING MODELS AND MONTE CARLO SIMULATION

The engineering models as we understand them are true complements to the economic models for the analysis of real options. That is, they provide solutions to valuation issues that the economic analyses as we know them cannot handle. On the other hand, they lack the strong theoretical underpinnings of the economic analysis. Engineering models provide insights and information that the economic models cannot, and vice versa. And the engineering models are in some circumstances able to communicate more effectively to decision makers. Consequently, we argue that a full analysis of an important valuation can often profitably use both what we label as the engineering and the economic approaches – as we do in our example application to the valuation for the New Songdo City project.

Engineering models represent a conceptually different approach to the valuation of projects than the financial models. They address different issues from different perspectives. There are many aspects to the contrast in approaches, as we discuss further on. For the moment, an immediate distinction is that whereas the economic model tends to apply “*at 30,000 feet*,” as we said

previously, engineering models can look at much more detail, consider the realities on the ground much more closely – perhaps they fly at 3,000 feet (but let’s not push the analogy too far).

This difference in perspective has important consequences for the analysis. Because engineering models are sensitive to important details, they cannot rely on the crucial simplifying assumptions that form the basis for mathematical solutions used in the economic real option models. To make this point explicitly, consider two features of engineering models. They generally consider that:

- uncertain processes generally change over time and commonly feature jumps. For example, the analysis might reflect the common assumption that demand for a service or product initially grows rapidly and then later more slowly as demand saturates. It might also factor in the likelihood of one or more sudden jumps as associated with changes in legislation (to impose environmental standards or carbon taxes, say, that would increase the value of an investment in “green” infrastructure) or in the structure of the marketplace (as by the creation of a free trade area or the arrival of a competitor); and
- projects frequently involve a variety of options that might be exercised in any order. Significant infrastructure projects involve options to expand locally (adding capacity to what exists), geographically (extending the network), and technologically (adopting new technologies). Just like moves on a chessboard, these options can be exercised in many ways, although some evidently must precede others.¹⁶

By contrast, economic models typically presume that the uncertainties stem from a stationary stochastic process and focus on implications of a single option. Such assumptions constitute the basis of the widely used lattice method for analysing options pioneered by Cox and Ross (1979).¹⁷ As the engineering models go beyond simplified situations that enable direct analytical solutions, they thus rely on “brute force” approaches – they develop solutions by looking at all the possibilities they can. This is why what we call the “engineering” approaches to valuation generally rely on Monte Carlo simulation.

While Monte Carlo simulation is a necessary feature of the engineering model of valuation, it is not sufficient. An approach to valuation can use simulation (see Hoesli et al 2006) without therefore being what we would call an engineering valuation. Monte Carlo simulation is just a tool that can be used in many contexts. It is simply a process that considers the set of possible outcomes for the range of possible scenarios, and then derives appropriate measures of value from these results. In other words, the engineering approach to valuation is not just a mathematical method. The crux of the engineering model lies in how it frames the problem of valuing projects.

¹⁶ For example, the New Songdo City project includes options to expand horizontally onto as yet undeveloped plots; options to expand locally, as for the convention center designed for possible modular expansion on its site; and options to change technologically, for example by shifting the water supply from reservoirs to desalination. Or consider investments in urban water supply, such as those of Hyflux, based in Singapore. The company delivers Build-Own-Operate plants worldwide. For any particular city it must consider that demand over the next generation will expand in some fashion (and thus could benefit from pre-building certain components of the system in advance of need, to enable easy future expansion), and it must also recognize that the technology and the energy cost of water purification may change substantially (which means that it should be open to new forms and location for future plants, as a switch from reservoirs to desalination implies a shift from facilities at higher altitudes to ones seaside).

¹⁷ The efficiency of the lattice model depends on the possibility that alternative paths of development recombine so that the number of states to be examined expands linearly with the number of periods instead of exponentially, that is (1, 2, 3, 4, 5...) instead of (1, 2, 4, 8, 16...). This is only possible if we posit path independence in terms of the outcomes associated with each path, and this implies that we can effectively only value a single option. Luenberger (1998) provides a good exposition of this approach.

The engineering model of value has three particular features that make it useful and interesting:

1. it builds upon the perspective of the investor/developer team that is designing and implementing a project. It does this by building on the spreadsheet models of cash flows that the development team will be using in any case. This provides both an easy link between the DCF analyses and option valuation as well as great transparency for the analysis, and thus easier acceptance by the users;
2. the engineering model pays particular attention to the specific idiosyncratic risks associated with specific features of any projects – issues that would be of less interest or even beyond the ken of investors in a portfolio of projects such as REITs. Notably, the engineering model considers the value of important design details associated with a project, its “*options in*” the design, as defined by Wang and de Neufville (2006). These are the various forms of flexibility built into a project – such as the extra strength built into a bridge so that it can be eventually be double-decked should that seem desirable¹⁸; and
3. the engineering model focuses on providing information on the distribution of the potential outcomes associated with optionality in a project. In addition to presenting a single value or price of an option, the model readily provides information on features such as the value-at-risk, the upside potential or value-at-gain, maximum and minimum outcome values, and so on.

We motivate and describe these perspectives in detail next, and then illustrate their application through a simple example project.

It is important to reiterate in this context that economic real options analysis does not have much traction for many developers of infrastructure. Many have simply not heard of the approach. And of those that have been exposed to the concept, many find the concepts and mathematics lacking in transparency – and thus have little confidence in the approach. If we wish to address these practitioners, to have access to this market, then we need to have some tools that address the valuation of options in a way that they can understand.

As noted, engineering models of valuation explicitly attempt to frame the issues from the perspective of the developers and similar decision-makers. For starters, they build on the financial spread sheets since these appear to constitute the *lingua franca* for ordinary financial analysis. We see how this approach has the significant advantages of establishing credibility and of providing transparency. Decision-makers can examine the assumptions in a form to which they are accustomed. They can include as much detail as they want. They can easily see the effect of

¹⁸ The use of flexibility in designing projects is becoming increasingly common, as detailed in de Neufville and Scholtes (2011). The George Washington Bridge in New York and the Ponte do 25 Abril in Lisbon were both originally built with the strength to carry a second deck, and both got theirs almost a generation later. Buildings are often built with the extra strength to add extra stories (Guma et al 2009), highways laid out with extra width to enable expansion or the installation of metro lines (as done for the access corridor to the Washington/Dulles airport), and in Japan JR East built some of its Shinkansen bullet train viaducts with the strength to receive not just one but two extra levels as necessary.

different assumptions by changing the size or timing of cash flows, “pressing the button,” to effect a recalculation of the spread sheet, so that different results appear almost immediately.¹⁹

From the perspective of the analysts, the use of spread sheets enables them to model a project in a useful level of detail, with all the requisite subtleties. They can project demand according to any trend they wish, and can easily incorporate any kind of jumps or other step changes, thus reflecting the possible opening of complementary projects, the enactment of new regulations, step changes in the fee structures and so on. The engineering approach can thus model projects as designers and decision makers in the real world actually perceive and deal with them.

The focus on the realities of particular projects further distinguishes the engineering models from the economic models of valuation. We can view this as an extension of the interest in communicating with the investor/developer teams organizing and deciding on projects. These project developers may have limited ability to diversify over a portfolio of projects, and may thus need to pay considerable attention to idiosyncratic risks.²⁰ And the realism is bolstered by focusing the analysis on a valuation framework that highlights a distribution of *ex post* outcome results, showing the decision maker not just the expected or most likely outcome but the nature and extent of the “tails” (upside and downside). Economics options models can also indicate ex post outcome distributions, but with less flexibility and customizability to the specific design and decision parameters of a given project, and we find that the simulation approach often renders results distributions that seem more credible or meaningful to decision makers. Engineering models, inherently based on simulation analysis readily fulfil this need as they routinely supply information on the distribution of the possible values associated with the optionality features of a project.

To illustrate the nature and use of engineering models of evaluation, we apply the approach here to a conceptually simple case study that illustrates its range and power. The case concerns the development of a multi-story parking garage. It was inspired by an actual situation, the development of a similar facility for the Bluewater Shopping Centre in Kent, England, which is one of the largest in the UK and in Europe. The interesting feature of this parking facility is that it was built to enable future vertical expansion. The developers/designers sized and built its foundations and columns strong enough to add on several extra floors. In our terminology, this parking facility included an “*option in*,” an option that only existed because the engineers had built this flexibility into the project. The option value of the parking garage of course depends in the first instance on the success of the entire Bluewater project: good growth makes it attractive for the owner of the project to add on extra floors.

The exposition of this case highlights the features of the engineering model of the valuation of flexibility in design. Full analytical details are available in de Neufville et al (2006). de Neufville and Scholtes (2011) provide extensive discussion of the subtleties of the use of the engineering model and its application to the case study of the parking garage.

¹⁹ Of course, spreadsheets also have their limitations. In some circumstances the particular nature or complexity of the design and operational decisions that need to be examined may require more specialized software tools. But many of these tools integrate with common “*lingua franca*” packages such as Excel®, and we find it most useful to attempt as much as possible to keep high level analysis and decision-oriented valuations at a level of simplicity and detail that can be modeled directly in Excel or similar spreadsheet products.

²⁰ Consider the case of the GMR group, based in India. In recent years it has been a leading developer of airports (greenfield new airport for Hyderabad, total reconstruction of New Delhi airport, active participation in the development of Istanbul/Sabiha Gokcen second airport). This portfolio of projects, large compared to competitors, is too small or targeted to represent much by way of diversification that would eliminate idiosyncratic risks in this business.

The engineering model of valuation starts with a spread sheet of the cash flows for the project. It does not require any special format. It builds on whatever arrangement that the project development team may be using. The common essential feature is that the spread sheet provides some sort of projection of demand for and revenues from the service as it evolves over time, together with a projection of investment and operating costs over the same period. For simplicity, our example supposes a 20-year leasehold and considers annual revenues and expenses, but in principle the engineering model considers as much detail as desired.

By building on the spread sheets used by the investor/developer team, the analysis immediately builds rapport with that group. There is no need to adopt new assumptions, to challenge the vision and perceptions of the developers. This approach furthermore builds in transparency in the process of valuing the optionality of the project. The project team can easily explore the implications of any of the special features of the project that they consider to be important, and that they have therefore already embedded into the spread sheet model.

We now need to consider carefully the three aspects that give rise to the value of the flexibility or optionality of a project. First there is the uncertainty itself: the risks and opportunities that may occur determine whether an option is exercised and thereby generates extra value for the project. Second, there is the range of decision choices or options available to the developers or operators of a system. The engineering model offers the analyst a great capacity to customize the decision choices, for example in the case of the parking garage we could specify that the owners could add either a single level at a time, or might be restricted to adding two at once. Third, there is the timing of exercising the option: it makes a difference when an option might be exercised, and we must address this issue carefully and realistically. The engineering model enables the analyst to represent both these features generally in greater detail than in an economic option value model. It therefore enables a much more realistic and accurate assessment of the opportunities.

When we represent the evolution of uncertainty in the spread sheet, we have basically no restrictions on the trend of this evolution or what we enter in any period for the demand, revenues, costs or whatever element of uncertainty we wish to model. For example, in the case of the parking garage we posited that the growth in demand over the first half of the project would increase exponentially as the Shopping Centre developed, but would then level off significantly as saturation set in. This is a standard kind of assumption for growth in demand but is of course only one possibility; other trend lines can be examined equally easily. It is also easy to impose jumps in the trends of any parameter, either explicitly at a specific time (as when it is known that a new lease agreement will take effect) or at some random time determined by the simulation process. In short, the spread sheet format permits us to analyse a wide range of forms for the distribution of uncertainty.

The greater facility of the engineering model to deal with realistic trends and disruptions, or any sort of custom-tailored random evolution in the relevant parameters, is particularly valuable at the more micro-level of the design and operation of physical components within an engineering system or a real estate or infrastructure asset. This is one reason why the approach is more appropriate for addressing “options *in*” such systems or assets, as distinct from options *on* them. In general a well-crafted engineering model pays little or no extra cost for dealing with whatever the analyst wishes to include. It deals with numbers in the cells equally, no matter how they got there.

The engineering model constructs a scenario over the life of the project year by year. Starting from the beginning, it defines the value of any of the parameters it is considering by drawing from the particular distribution assigned by the analyst to that year. This distribution may be a simple trend with a standard deviation; it may be a combination of a number of factors, such as the probability of

population growth and the probability of an economic recession; or some other factor. In this way it can develop literally thousands of scenarios very quickly. In our case, the analysis of the performance of the parking garage over 20 periods for 10,000 scenarios takes just a few seconds on a laptop. In the engineering model, the process of exploring the effects of uncertainty is fast and reliable.

Turning now to the matter of exercising options embedded in the model, the engineering model mimics the anticipated decision processes of the project managers over the lifetime of the project. That is, based on what the analysts and the investor/developer team decide would or should be the behaviour of managers over time, the engineering model embeds “decision rules” in the spread sheet. For instance, a rule we adopted for the parking facility was “add an extra floor to the garage if demand exceeded capacity over the preceding two years” – the idea being that managers would not react too quickly to spikes in demand, but might delay their response until the traffic growth appeared to be established. Note that this approach is essentially descriptive – it does not pretend to be optimal. It should be developed by consultation with the actual decision makers in the real world. To implement the decision rules that trigger the exercise of options, the engineering model embeds them as “if” statements in appropriate cells of the spread sheet. Thus as the simulation unfolds a scenario period by period, in each period an “if” statement checks to see if the evolution of the parameter (the demand for example) has satisfied the condition requiring the exercise of the option. If the condition is met, the “if” statement triggers the action, making the associated changes (such as increasing the capacity of the garage, and incurring the cost of the investment associated with the expansion).

Note that in this arrangement there is no requirement that the decision rule be constant over the entire life of the project. Indeed, our analysis of the garage case embedded different “if” statements for different periods. For example, one version required greater growth to trigger expansion in the last years of the project, another even closed off the option (it might not be sensible to expand the facility in the last years of the leasehold). In short, here again the engineering model offers great flexibility in describing the potential evolution of the project and assists in thinking about management policies.

The engineering model adopts this descriptive approach to specifying the exercise of options out of necessity. The possible complexity of a realistic design and decision situation with the evolution of the relevant parameters and factors precludes optimization in a formal or rigorous sense, but is also not limited by the simplifying assumptions necessary to derive such formal optimality. Instead, the engineering model enables the analyst to mimic actual real world behaviour closely. Further, as simulation models do not have to presume path independence for outcomes, the engineering model of valuation can consider multiple options exercised over the period of analysis. The result is a reasonable approach that can deal with multiple options under all kinds of circumstances.

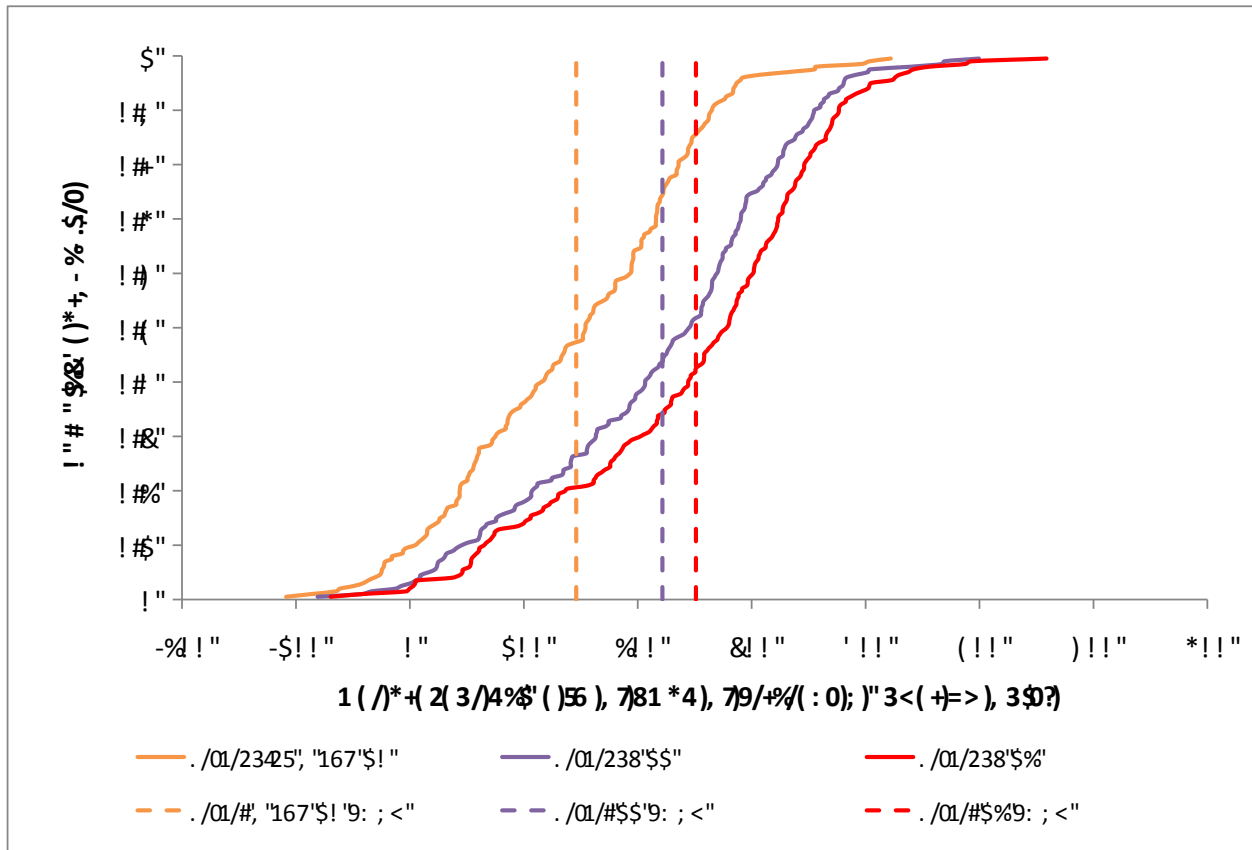
The results coming from the engineering model of value consist of a distribution of possible “future values” of a project with optionality. Each scenario for the combination of uncertainties leads to a set of possible exercises of the options and thus a value for the project derived in the usual way from the associated cash flow. The resulting values are effectively *ex post* values, the outcomes of particular “future histories” generated in the simulation. However, a useful metric for quantifying these *ex post* results is often a “net present value” (NPV) computation, that is, a discounting of the *ex post* results back to a valuation as of the time of inception of the project.²¹

²¹ One could as easily employ a “net terminal value” metric, but NPV is the more common and widely accepted perspective.

Combining these individual results, for however many thousand simulations the analysis involves, gives the distribution of project results. These can be summarized in terms of their central tendency by the mean of that *ex post* outcome distribution, a sort of expected value to represent a “value with options” for the project. However, in our experience, the distribution of the outcome values is often more interesting than its average or central tendency. Indeed, the expected value of the project may easily not be usefully representative of its economic results. In general, the distribution may be skewed with fat tails on the downside or upside. In either case, the average value may be unrepresentative of the median or modal possible outcomes. For example, investments in start-up IT companies have led to very impressive returns (Microsoft, Google, Facebook and so on) but the vast majority of them are failures in which the investors “lose their shirts”. Insofar as investor/developer teams care about the distribution of possible returns (either because they are risk averse or because they are on the contrary willing to bet on possible extraordinary gains) then the engineering model of valuation has the merit of focusing conveniently on the distribution of possible values of a project.

At this point it is important to stress that engineering models can have an important role in uncovering significant sources of value – and thus of greatly improving design. Here’s how it works: the exploration of the value of various possible flexibilities in the design – that is, of options – leads to the identification of sources of value that the designers did not originally recognize. Indeed, designers often, perhaps habitually, focus on creating the details of a project that clients have specified to them. Designers do not usually challenge the specifications, and therefore may not go out of their way to explore alternative specifications for a project. When we use engineering models to value flexible designs that could fulfil alternative specifications, we frequently uncover flexible design alternatives that could provide much greater value, and therefore lead the design team to significantly more valuable configurations of a project. In this regard, there is a rich and varied experience with engineering models of value for projects with optionality in all kinds of investments in infrastructure, in the natural resource extraction industry (oil, gas, minerals), in major industries (automobile manufacture, satellite development), and in product development. We have shared in many such cases and can report that the use of the engineering model often uncovers significant sources of value.

A salient example of our experience with engineering models is the work we did recently with BP. This relates to the design of their offshore development of infrastructure for one of their oil fields, as reported by Lin *et al* 2011. The case involved the choice of the design and the location of a series of oil platforms and the associated sub-sea connections between wellheads. The total cost of this infrastructure would easily exceed US\$5 billion, depending on what BP eventually decides to build, on what options they choose to exercise. The major uncertainties around the project concern the price of oil; the quality of the oil, specifically as regards its viscosity, which determines its flow; and ultimately on the amount of oil that is economic to recover, which depends on the price of oil, its quality, and also on the geology of the field. The analysis considered both “*options on*” and “*options in*” the project. The former involved decisions about when to exercise the option to install additional platforms, the latter the ability to connect the platforms with different parts of the oil field, as determined by the design of the sub-sea connections. The analysis used a spread sheet backed by extensive models of the performance of the system for recovering oil. That is, we obtained the costs and revenues in each cell for each period by calling upon a reasonably detailed petroleum engineering model of the complex system associated with the recovery of oil and gas.



Results from Application of Engineering Model to Development of Infrastructure for an Oil Field
Source: Authors
Exhibit 3

By using the engineering model for the valuation of options, we were able to uncover several designs for the system that had great value and which had not been previously considered. Exhibit 3 illustrates the results. Each curve represents the target curve, or cumulative distribution of the possible values of different design strategies. These curves came directly from the application of the engineering option model to a particular design. The simulation analysis for each design indicated the range and distribution of possible outcome values, and permitted us to calculate an expected value. The “option value” associated with any design is then the difference between its expected value and that of a design without options. Given that each design involves many different options, and that managers would exercise different ones depending on the scenario, the “option value” for any design cannot be associated with any single option, it is a value ascribed to a set of options. The curves in Exhibit 3 indicate the performance of the three best alternative designs that our team uncovered. The curve furthest to the right offers the best set of outcomes and should be preferred. While the overall level of benefits is confidential, Exhibit 3 indicates about 150% increase in value on a multi-billion dollar project which is tremendous.

Editor’s Note: Professors Geltner and de Neufville’s substantial paper has been divided into two parts. The first part (Vol 18, No 3, pps 231-249) includes the introduction and consideration of economic real option models, engineering models and Monte Carlo simulation. The second part (Vol 18, No 3, pps 251-276) includes a consideration of quantifying uncertainty or volatility

through real asset pricing data and indexing, an application to Songdo New City and conclusions. For ease of reference, the abstract and references have been included in both parts.

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Email contact: dgeltner@MIT.EDU